Province of the
EASTERN CAPE
EDUCATION

CURRICULUM FET

MATHEMATICS

REVISION MATERIAL

#MASTER THE BASICS FIRST

Booklet 1 of 2018:

- Equations and Inequalities
- Sequences and Series
- Functions and Inverses
- Calculus
- Statistics
- Analytical Geometry

It does not matter what percentage you got in the past, you can collect a lot of marks if you focus and practice the topics above!

Practice makes perfect! Collect the marks!

RESOURCE MATERIAL
BOOKLET 1 OF 2018

1. ALGEBRA (EQUATIONS AND INEQUALITIES / NATURE OF THE ROOTS) [25 ±3MARKS]

- Solve for x: Factorization, using the Quadratic Formula
- Inequalities
- Simultaneous Equation
- Nature of the roots
- Unseen general question, understanding function notation, exponents, surds, etc

2. ANALYTICAL GEOMETRY [40±3 MARKS]

- Straight line: Length, gradient, Midpoint, bisect, parallel, perpendicular
- Characteristics of triangles(isosceles; equilateral) and quadrilaterals(rectangle; square; rhombus; etc)
- Equation of Straight Line, Angle of Inclination
- Circle: Equation (centre at origin / centre at any point)
- Changing between forms of circle equations ; Equation of tangent ; etc
- Points of intersection

3. NUMBER PATTERNS / SEQUENCES & SERIES [25 ±3MARKS]

- Understanding patterns
- General Term
- Value of n / number of terms
- Arithmetic & Geometric Sequences and Series
- Sum to infinity
- Use of Formulae
  - Find n^{th} term / Find Sum of *n* terms / Find the values of *a*, *d/r* using simultaneous equations
  - Finding an unknown in a given sequence. Arithmetic, Geometric or Quadratic
- Word problems

4. FUNCTIONS AND GRAPHS [35±3 MARKS]

- Straight line / Parabola / Hyperbola/ Exponential
- Sketching
- Interpretation
- Combinations of graphs on same system of axes
- Transformations
- Inverses
- Etc

5. DATA HANDLING [20±3 MARKS]

- Mean, Median, Mode, Range, Standard Deviation, Describe questions
- Scatter plot, Outliers, Reading from graph – in both directions, Predictions, Table, etc
- Cumulative Frequency, Ogives, Lower and Upper Quartile, Box–and–whisker, Comment on , etc
- Regression and Correlation : least squares regression line / Estimations / Symmetric /Skewness
- Histograms, Frequency Polygons

6. CALCULUS [35±3 MARKS]

- Limits
- Derivative: First Principles/Laws
- Average gradient
- Application: 3^{rd} degree graph sketching including second derivative for point of inflection
- Application: Optimization

REMEMBER: PRACTICE MAKES PERFECT!!!! HARDWORK NEVER KILLS!!!!

SO, PRACTICE!!! PRACTICE!!!! PRACTICE!!!!!

EQUATIONS AND INEQUALITIES

QUESTION 1

1.1 Solve for x, rounded off to TWO decimal places where necessary:

1.1.1 $x^2 = 5x - 4$ (3)

1.1.2 $x(3-x) = -3$ (5)

1.1.3 $3 - x < 2x^2$ (5)

1.2 Determine the values of x and y if they satisfy both the following equations simultaneously:

$$2x + y = 3$$
$$x^2 + y + x = y^2$$ (8)

1.3 Given $x = 999\,999\,999\,999$, determine the exact value of $\frac{x^2-4}{x-2}$.

Show ALL your calculations. (3)

1.4 Explain why the equation $\frac{x^4+1}{x^4} = \frac{1}{2}$ has no real roots. (2)

[26]

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-1) = 30$ (3)

1.1.2 $3x^2 - 5x + 1 = 0$ (Correct to ONE decimal place) (4)

1.1.3 $15x - 4 < 9x^2$ (4)

1.2 Solve simultaneously for x and y in the following set of equations:

$$x - y = 3$$
$$x^2 - xy - 2y^2 - 7 = 0$$ (5)

1.3 Calculate the exact value of:

$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}}$$ (Show ALL calculations.) (3)

1.4 Simplify completely without the use of a calculator:

$$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$$ (3)

[22]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(x+5)=9$ (4)

1.1.2 $2x^2-2\leq 3x$ (4)

1.2 Solve simultaneously for x and y:

$$2+y=-2x$$
$$-2x^2+8xy+42=y$$ (7)

1.3 If $f(x)=\sqrt{4x}$ and $g(x)=x^2$, determine $f(g(9))$. (3)

1.4 If $\frac{14}{\sqrt{63}-\sqrt{28}}=a\sqrt{b}$, determine, without using a calculator, the value(s) of a and b if a and b are integers. (4)

[22]

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2-5x=2$ (3)

1.1.2 $x-\frac{2}{x}=5$ (4)

1.1.3 $(x+1)(x-3)>12$ (4)

1.2 Solve simultaneously for r and p in the following set of equations:

$$6r+5rp-5p=8$$
$$r+p=2$$ (7)

1.3 The volume of a box with a rectangular base is 3 072 cm^3. The lengths of the sides are in the ratio 1 : 2 : 3. Calculate the length of the shortest side. (4)

[22]

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $(2x-1)(x+4)=0$ (2)

1.1.2 $3x^2-x=5$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^2+7x-8<0$ (4)

1.2 Given: $4y-x=4$ and $xy=8$

1.2.1 Solve for x and y simultaneously. (6)

1.2.2 The graph of $4y-x=4$ is reflected across the line having equation $y=x$. What is the equation of the reflected line? (2)

1.3 The solutions of a quadratic equation are given by $x=\dfrac{-2\pm\sqrt{2p+5}}{7}$

For which value(s) of p will this equation have:

1.3.1 Two equal solutions (2)

1.3.2 No real solutions (1)

[21]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x^2-9)(2x+1)=0$ (3)

1.1.2 $x^2+x-13=0$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $2.3^x=81-3^x$ (4)

1.1.4 $(x+1)(4-x)>0$ (3)

1.2 Given: $2^x+2^{x+2}=-5y+20$

1.2.1 Express 2^x in terms of y. (2)

1.2.2 How many solutions for x will the equation have if $y=-4$? (2)

1.2.3 Solve for x if y is the largest possible integer value for which $2^x+2^{x+2}=-5y+20$ will have solutions. (3)

[21]

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-2)(4+x)=0$ (2)

1.1.2 $3x^2-2x=14$ (correct to TWO decimal places) (4)

1.1.3 $2^{x+2}+2^x=20$ (3)

1.2 Solve the following equations simultaneously:

$x=2y+3$

$3x^2-5xy=24+16y$ (6)

1.3 Solve for x: $(x-1)(x-2)<6$ (4)

1.4 The roots of a quadratic equation are: $x=\dfrac{3\pm\sqrt{-k-4}}{2}$

For which values of k are the roots real? (2)

[21]

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2-x-12=0$ (3)

1.1.2 $x(x+3)-1=0$ (Leave your answer in simplest surd form.) (3)

1.1.3 $x(4-x)<0$ (3)

1.1.4 $x=\dfrac{a^2+a-2}{a-1}$ if $a=888\ 888\ 888\ 888$ (2)

1.2 Solve the following equations simultaneously:

$y+7=2x$ and $x^2-xy+3y^2=15$ (6)

1.3 Determine the range of the function $y=x+\dfrac{1}{x}$, $x\neq 0$ and x is real. (6)

[23]

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x+1)=6$ (3)

1.1.2 $3x^2-4x=8$ (4)

1.1.3 $4x^2+1\geq 5x$ (4)

1.2 Consider the equation: $x^2+5xy+6y^2=0$

1.2.1 Calculate the values of the ratio $\frac{x}{y}$. (3)

1.2.2 Hence, calculate the values of x and y if $x+y=8$. (5)
[19]

SEQUENCES AND SERIES

QUESTION 3

The following is an arithmetic sequence:

$$1-p \;;\; 2p-3 \;;\; p+5 \;;\; \ldots$$

3.1 Calculate the value of p. (3)

3.2 Write down the value of:

3.2.1 The first term of the sequence (1)

3.2.2 The common difference (1)

3.3 Explain why none of the numbers in this arithmetic sequence are perfect squares. (2)
[7]

QUESTION 4

Consider the sequence: $6 \;;\; 6 \;;\; 2 \;;\; -6 \;;\; -18 \;;\; \ldots$

4.1 Write down the next term of the sequence, if the sequence behaves consistently. (1)

4.2 Determine an expression for the n^{th} term, T_n. (5)

4.3 Show that -6838 is in this sequence. (4)
[10]

QUESTION 3

Given: $\sum_{t=0}^{99}(3t-1)$

3.1 Write down the first THREE terms of the series. (1)

3.2 Calculate the sum of the series. (4)

[5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$$-3\ ;-2\ ;-3\ ;-6\ ;-11\ ;\ \ldots$$

4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

4.2 Calculate the first difference between the 35^{th} and 36^{th} terms of the quadratic sequence. (2)

4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

4.4 Explain why the sequence of numbers will never contain a positive term. (2)

[11]

2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: 5 ; 9 ; 13 ; 17 ; 21 ; …
Tebogo's sequence: 5 ; 125 ; 3 125 ; 78 125 ; 1 953 125 ; …

Write down the n^{th} term (or the rule in terms of n) of:

2.1.1 Matthew's sequence (3)

2.1.2 Tebogo's sequence (2)

2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)

[10]

QUESTION 2

Consider the following sequence: 399 ; 360 ; 323 ; 288 ; 255 ; 224 ; …

2.1 Determine the n^{th} term T_n in terms of n. (6)

2.2 Determine which term (or terms) has a value of 0. (3)

2.3 Which term in the sequence will have the lowest value? (1)

[10]

QUESTION 3

3.1 Prove that: $a + ar + ar^2 + \ldots$ (to n terms) $= \dfrac{a(r^n - 1)}{r - 1}$, $r \neq 1$ (4)

3.2 Given the geometric series: $3 + 1 + \dfrac{1}{3} + \ldots$

Calculate the sum to infinity. (3)

[7]

QUESTION 2

The sequence 3 ; 9 ; 17 ; 27 ; … is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the n^{th} term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

QUESTION 3

3.1 The first two terms of an infinite geometric sequence are 8 and $\dfrac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16 + 8\sqrt{2}$. (4)

3.2 The following geometric series is given: $x = 5 + 15 + 45 + \ldots$ to 20 terms.

3.2.1 Write the series in sigma notation. (2)

3.2.2 Calculate the value of x. (3)

[9]

QUESTION 4

4.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n+9)$

4.1.1 Calculate the sum to 23 terms of the sequence. (2)

4.1.2 Hence calculate the 23rd term of the sequence. (3)

4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.

Determine TWO possible values for the common ratio, r, of the geometric sequence. (6)

[11]

QUESTION 2

2.1 Given the sequence: $4\ ;\ x\ ;\ 32$

Determine the value(s) of x if the sequence is:

2.1.1 Arithmetic (2)

2.1.2 Geometric (3)

2.2 Determine the value of P if $\mathrm{P} = \sum_{k=1}^{13} 3^{k-5}$ (4)

2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d, the sum to n terms can be expressed as $S_n = \frac{n}{2}(2a+(n-1)d)$. (4)

[13]

QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3\ ;\ 3\ ;\ 9\ ;\ 6\ ;\ 15\ ;\ 12\ ;\ \ldots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52} - T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)

[9]

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to – 6 and a fifth term equal to – 14.

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)
[7]

QUESTION 2

Given the arithmetic series: $-7-3+1+\ldots+173$

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)
[9]

QUESTION 3

3.1 Consider the geometric sequence: $4\ ;\ -2\ ;\ 1\ \ldots$

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series $4-2+1\ \ldots$ (2)

3.2 If x is a REAL number, show that the following sequence can NOT be geometric:

$1\ ;\ x+1\ ;\ x-3\ \ldots$ (4)
[12]

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100 - 3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

QUESTION 3

Consider the infinite geometric series: $45 + 40{,}5 + 36{,}45 + \ldots$

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

QUESTION 2

2.1 Given the following quadratic sequence: $-2\ ;\ 0\ ;\ 3\ ;\ 7\ ;\ ...$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)
[18]

QUESTION 2

Given the geometric sequence: $-\frac{1}{4}\ ;\ b\ ;\ -1\ ;\ .....$

2.1 Calculate the possible values of b. (3)

2.2 If $b=\frac{1}{2}$, calculate the 19^{th} term (T_{19}) of the sequence. (3)

2.3 If $b=\frac{1}{2}$, write the sum of the first 20 positive terms of the sequence in sigma notation. (4)

2.4 Is the geometric series formed in QUESTION 2.3 convergent? Give reasons for your answer. (2)
[12]

QUESTION 3

3.1 $6\ ;\ 6\ ;\ 9\ ;\ 15\ ;\ ...$ are the first four terms of a quadratic number pattern.

3.1.1 Write down the value of the fifth term (T_5) of the pattern. (1)

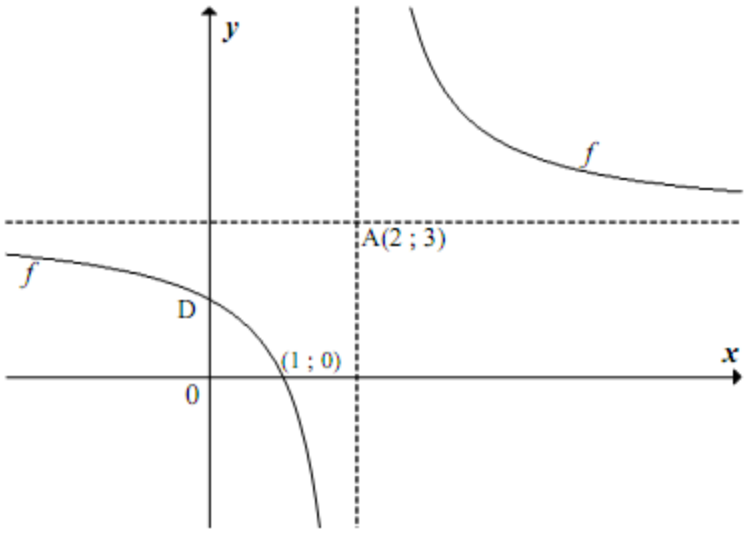
3.1.2 Determine a formula to represent the general term of the pattern. (4)

3.1.3 Which term of the pattern has a value of 3 249? (4)

FUNCTIONS AND INVERSES

QUESTION 4

Given $f(x) = \frac{a}{x-p} + q$. The point A(2 ; 3) is the point of intersection of the asymptotes of f.
The graph of f intersects the x-axis at (1 ; 0).
D is the y-intercept of f.



4.1 Write down the equations of the asymptotes of f. (2)

4.2 Determine an equation of f. (3)

4.3 Write down the coordinates of D. (2)

4.4 Write down an equation of g if g is the straight line joining A and D. (3)

4.5 Write down the coordinates of the other point of intersection of f and g. (4)
[14]

QUESTION 5

Consider the function $f(x) = 4^{-x} - 2$.

5.1 Calculate the coordinates of the intercepts of f with the axes. (4)

5.2 Write down the equation of the asymptote of f. (1)

5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)

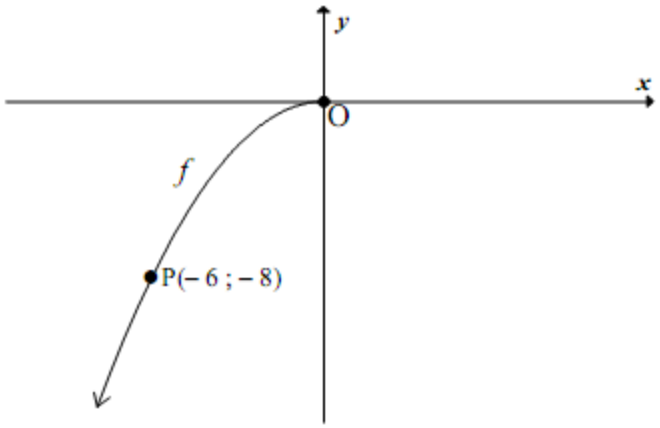
5.4 Write down the equation of g if g is the graph of f shifted 2 units upwards. (1)

5.5 Solve for x if $f(x) = 3$. (You need not simplify your answer.) (3)

[12]

QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.
The point P$(-6 ; -8)$ lies on the graph of f.



6.1 Calculate the value of a. (2)

6.2 Determine the equation of f^{-1}, in the form $y = \ldots$ (3)

6.3 Write down the range of f^{-1}. (1)

6.4 Draw the graph of f^{-1} on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from $(0 ; 0)$. (2)

6.5 The graph of f is reflected across the line $y = x$ and thereafter it is reflected across the x-axis. Determine the equation of the new function in the form $y = \ldots$ (3)

[11]

QUESTION 5

Consider the function $f(x) = \frac{3}{x-1} - 2$.

5.1 Write down the equations of the asymptotes of f. (2)

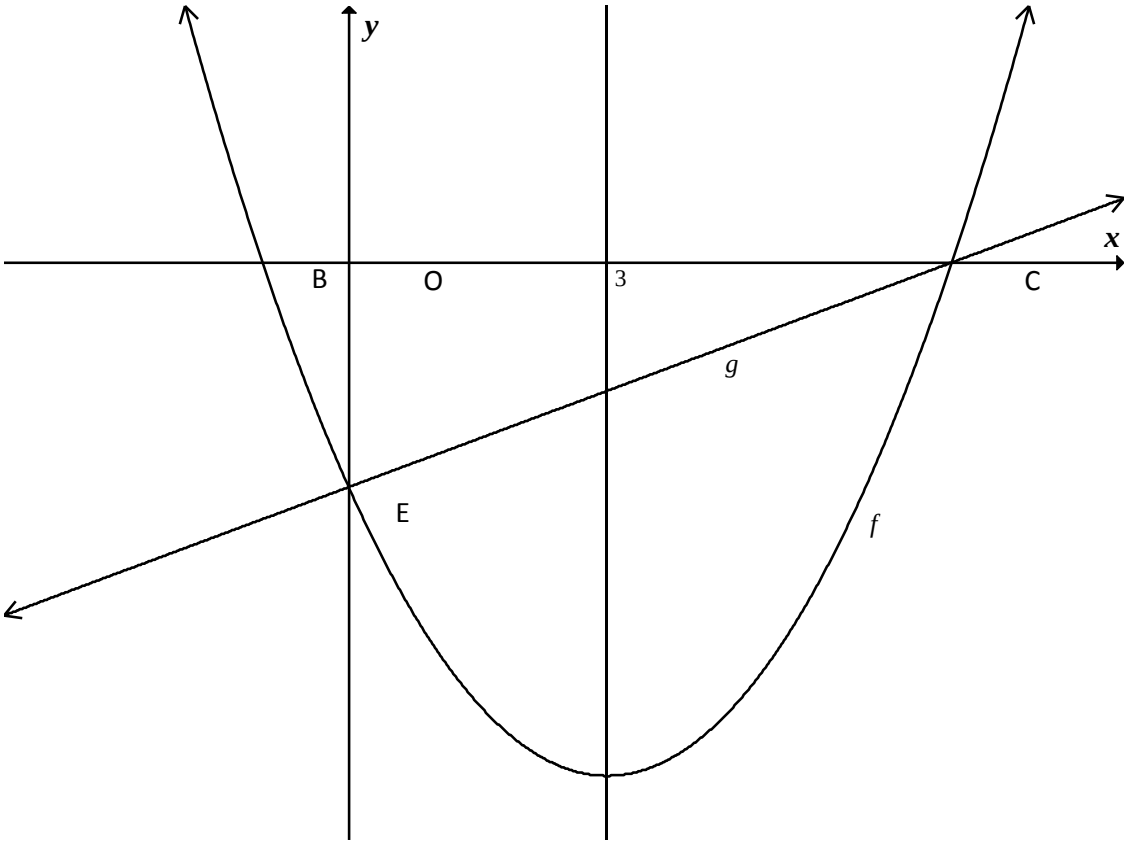
5.2 Calculate the intercepts of the graph of f with the axes. (3)

5.3 Sketch the graph of f on DIAGRAM SHEET 1. (3)

5.4 Write down the range of $y = -f(x)$. (1)

QUESTION 6

A parabola f intersects the x-axis at B and C and the y-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



6.1 Show that the coordinates of C are (7 ; 0). (1)

6.2 Calculate the x-coordinate of B. (1)

6.3 Determine the equation of f in the form $y = a(x-p)^2 + q$. (6)

6.4 Write down the equation of the graph of h, the reflection of f in the x-axis. (1)

QUESTION 7

Consider the function $f(x)=\left(\frac{1}{3}\right)^x$.

7.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

7.2 Determine $f^{-1}(x)$ in the form $y=\ldots$ (2)

7.3 Write down the equation of the asymptote of $f(x)-5$. (1)

7.4 Describe the transformation from f to g if $g(x)=\log_3 x$. (2)

[7]

QUESTION 5

5.1 Consider the function: $f(x)=\frac{-6}{x-3}-1$

5.1.1 Calculate the coordinates of the y-intercept of f. (2)

5.1.2 Calculate the coordinates of the x-intercept of f. (3)

5.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

QUESTION 6

Given: $f(x)=3^x$

6.1 Determine an equation for f^{-1} in the form $f^{-1}(x)=\ldots$ (1)

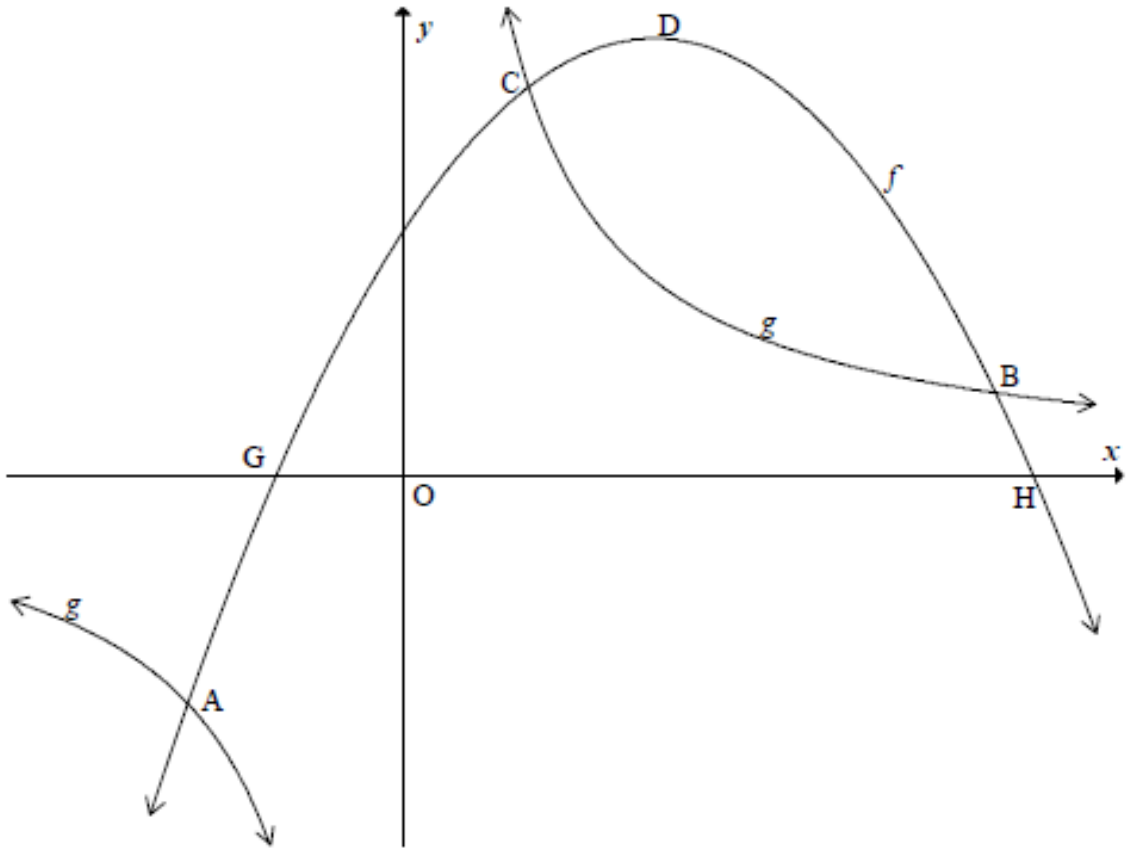
6.2 Sketch, in your ANSWER BOOK, the graphs of f and f^{-1}, showing clearly ALL intercepts with the axes. (4)

6.3 Write down the domain of f^{-1}. (2)

QUESTION 5

The graphs of the functions $f(x) = -2x^2 + 8x + 10$ and $g(x) = \frac{16}{x}$ are sketched below.

G and H are the x-intercepts of f.
D is the turning point of f.
Points A, B and C are the points of intersection of f and g.



5.1 Write down the equations of the asymptotes of the graph of g. (2)

5.2 Determine the coordinates of H. (4)

5.3 Determine the range of f. (4)

5.4 Verify that C is the point $(1 ; 16)$. (2)

5.5 Determine the coordinates of the turning point of p if $p(x) = f(3x)$. (3)
[15]

QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

4.1.1 Calculate the coordinates of the y-intercept of the graph of f. (1)

4.1.2 Calculate the coordinates of the x-intercept of the graph of f. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes. (3)

4.1.4 Write down the range of f. (1)

QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

4.1.1 Calculate the coordinates of the y-intercept of the graph of f. (1)

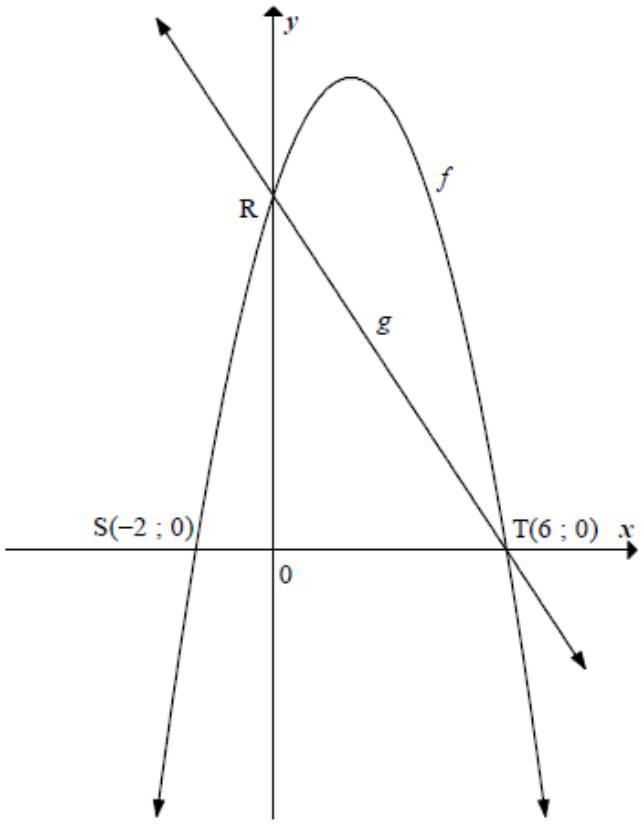
4.1.2 Calculate the coordinates of the x-intercept of the graph of f. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes. (3)

4.1.4 Write down the range of f. (1)

4.2 S(−2 ; 0) and T(6 ; 0) are the x-intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the y-intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



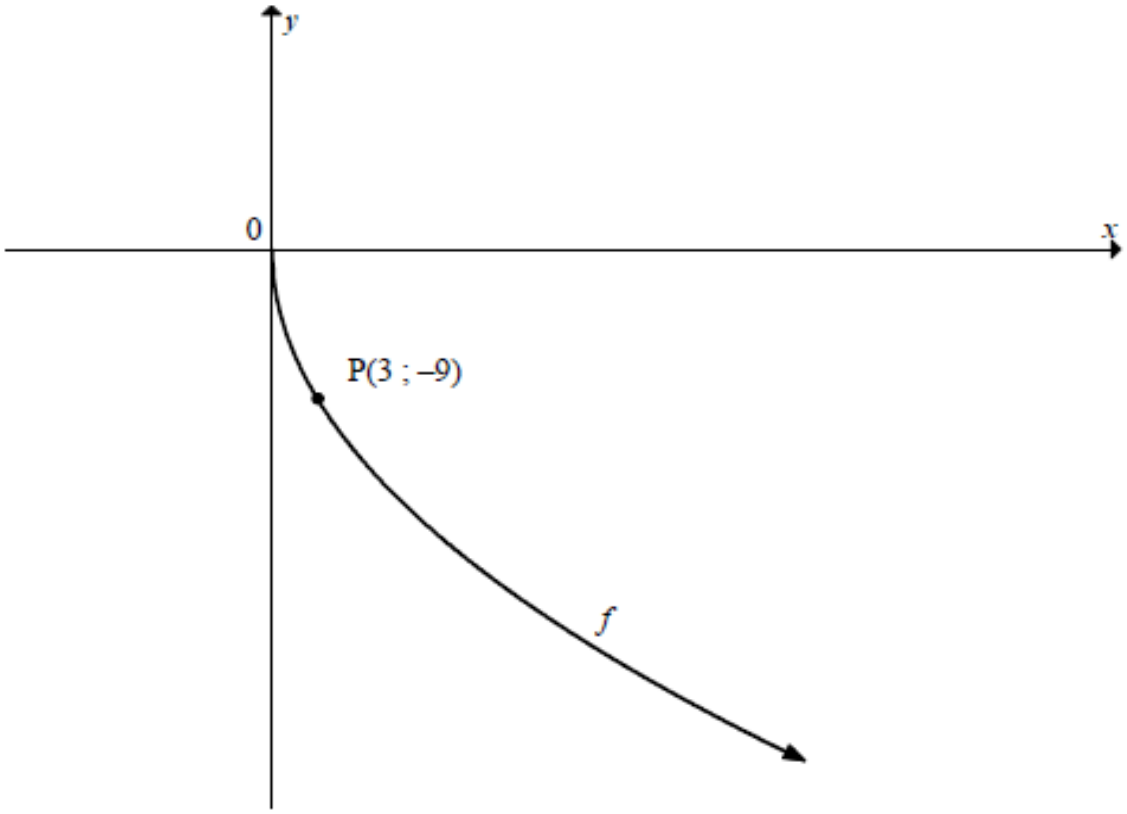
4.2.1 Determine the value of d. (2)

4.2.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$. (4)

4.2.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of f. (2)

QUESTION 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of f.



5.1 Use your graph to determine the values of x for which $f(x) \geq -9$. (2)

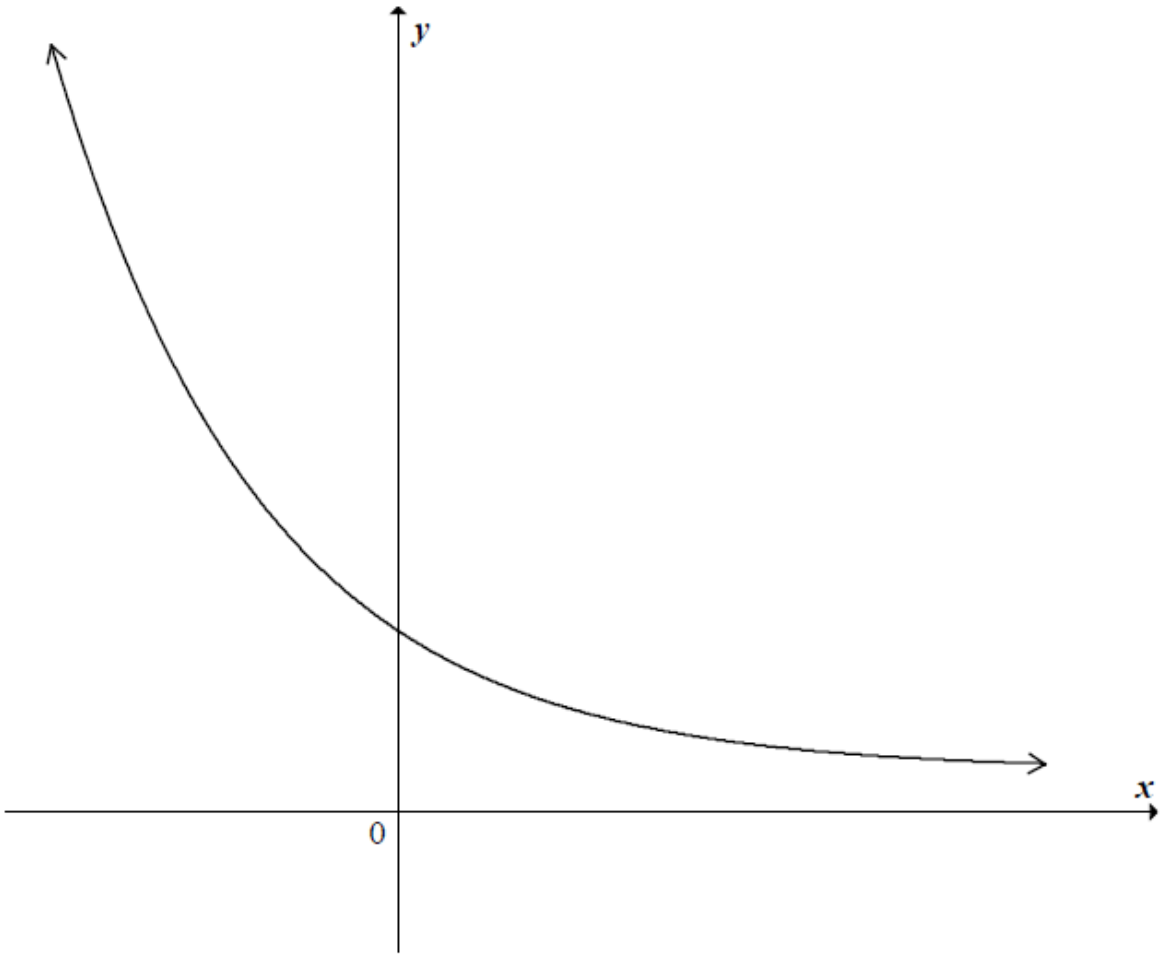
5.2 Write down the equation of f^{-1} in the form $y = \ldots$ Include ALL restrictions. (3)

5.3 Sketch f^{-1}, the inverse of f, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

5.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)
[9]

QUESTION 4

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



4.1 Write down the domain of f. (1)

4.2 Write down the equation of the asymptote of f. (1)

4.3 Write down the equation of f^{-1} in the form $y = ...$ (2)

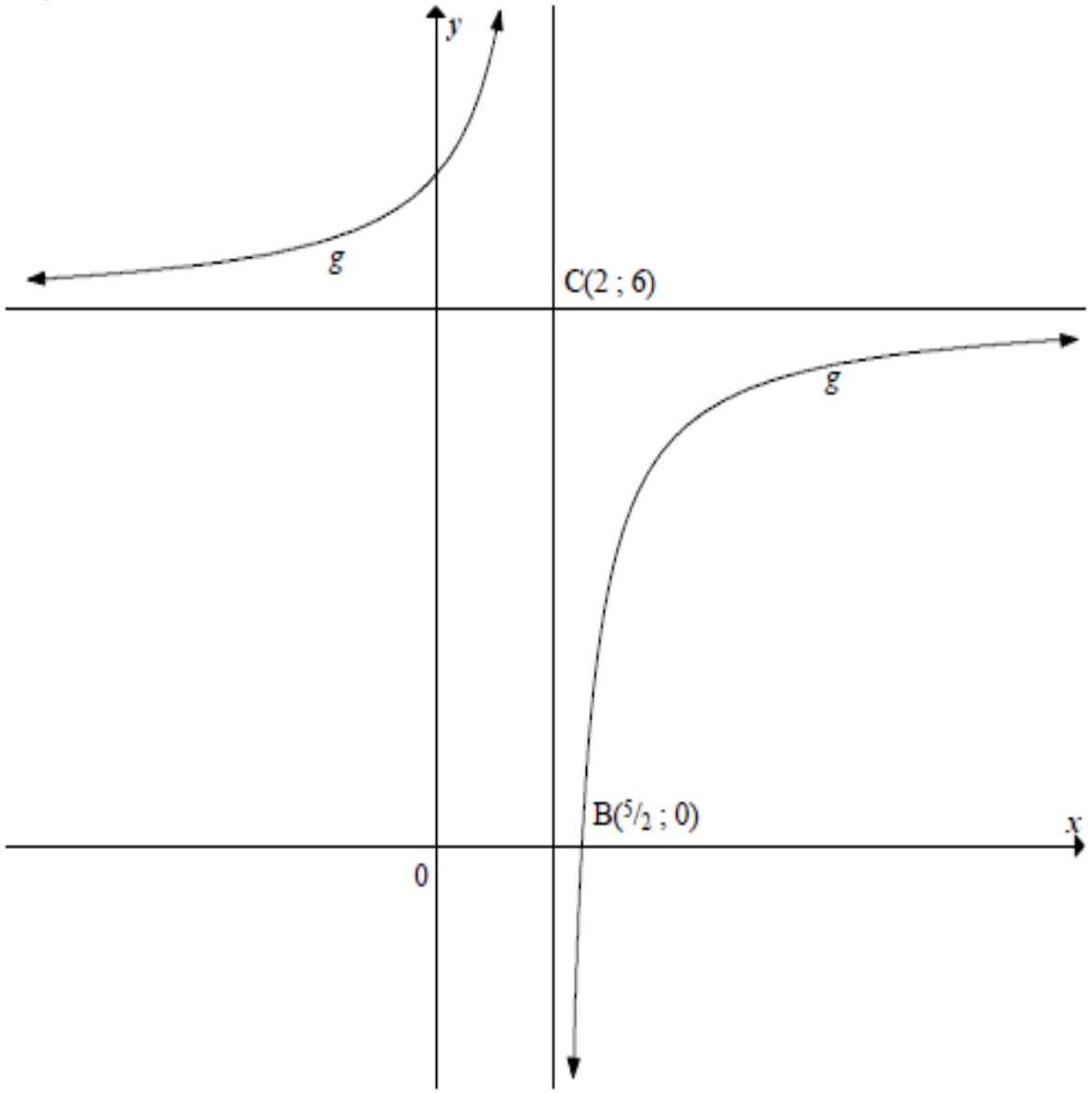
4.4 Sketch the graph of f^{-1} in your ANSWER BOOK. Indicate the x-intercept and ONE other point. (3)

QUESTION 5

Sketched below is the graph of $g(x) = \frac{a}{x-p} + q$.

C(2 ; 6) is the point of intersection of the asymptotes of g.

B$\left(\frac{5}{2}; 0\right)$ is the x-intercept of g.



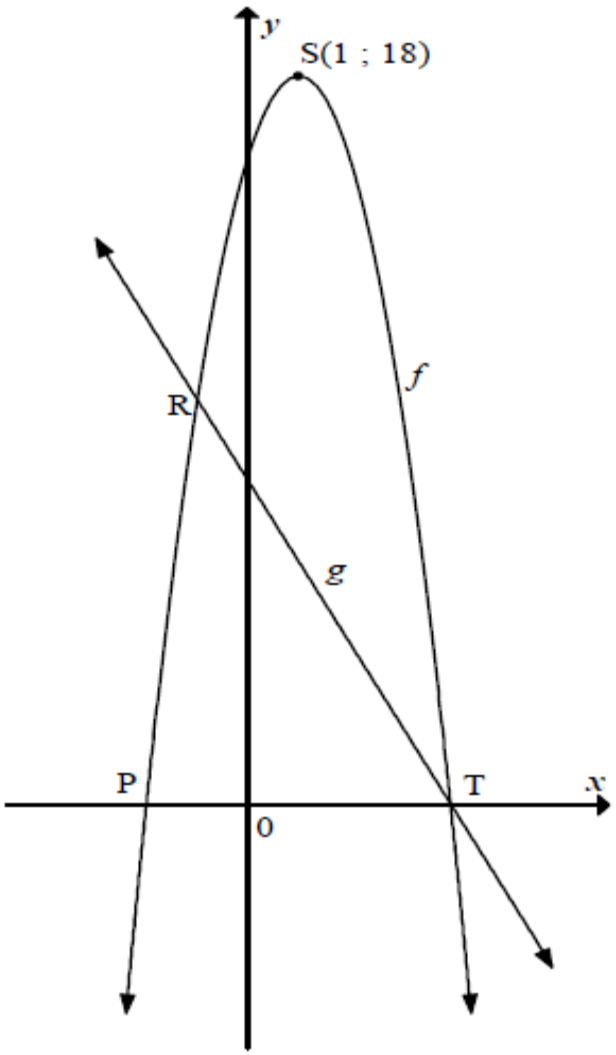
5.1 Determine the equation for g in the form $g(x) = \frac{a}{x-p} + q$ (4)

5.2 F is the reflection of B across C. Determine the coordinates of F. (2)

[6]

QUESTION 6

S(1 ; 18) is the turning point of the graph of $f(x) = ax^2 + bx + c$. P and T are x-intercepts of f. The graph of $g(x) = -2x + 8$ has an x-intercept at T. R is a point of intersection of f and g.



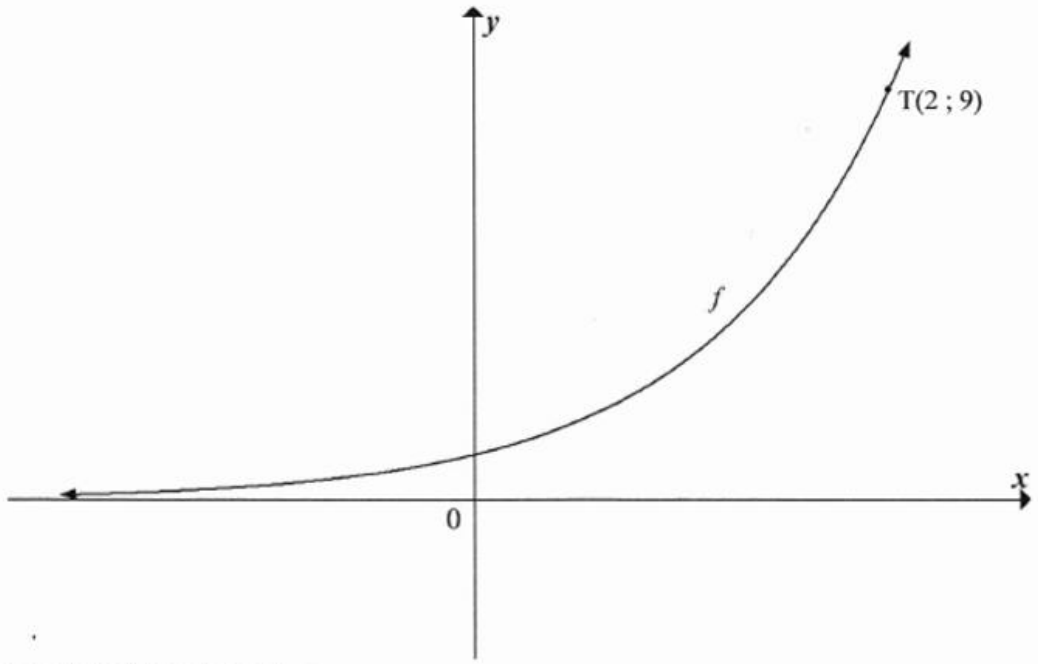
6.1 Calculate the coordinates of T. (2)

6.2 Determine the equation for f in the form $f(x) = ax^2 + bx + c$. Show ALL your working. (4)

6.3 If $f(x) = -2x^2 + 4x + 16$, calculate the coordinates of R. (4)

QUESTION 5

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)

5.3 Is the inverse of f a function? Explain your answer. (2)

QUESTION 4

Given: $f(x) = 2^{-x} + 1$

4.1 Determine the coordinates of the y-intercept of f. (1)

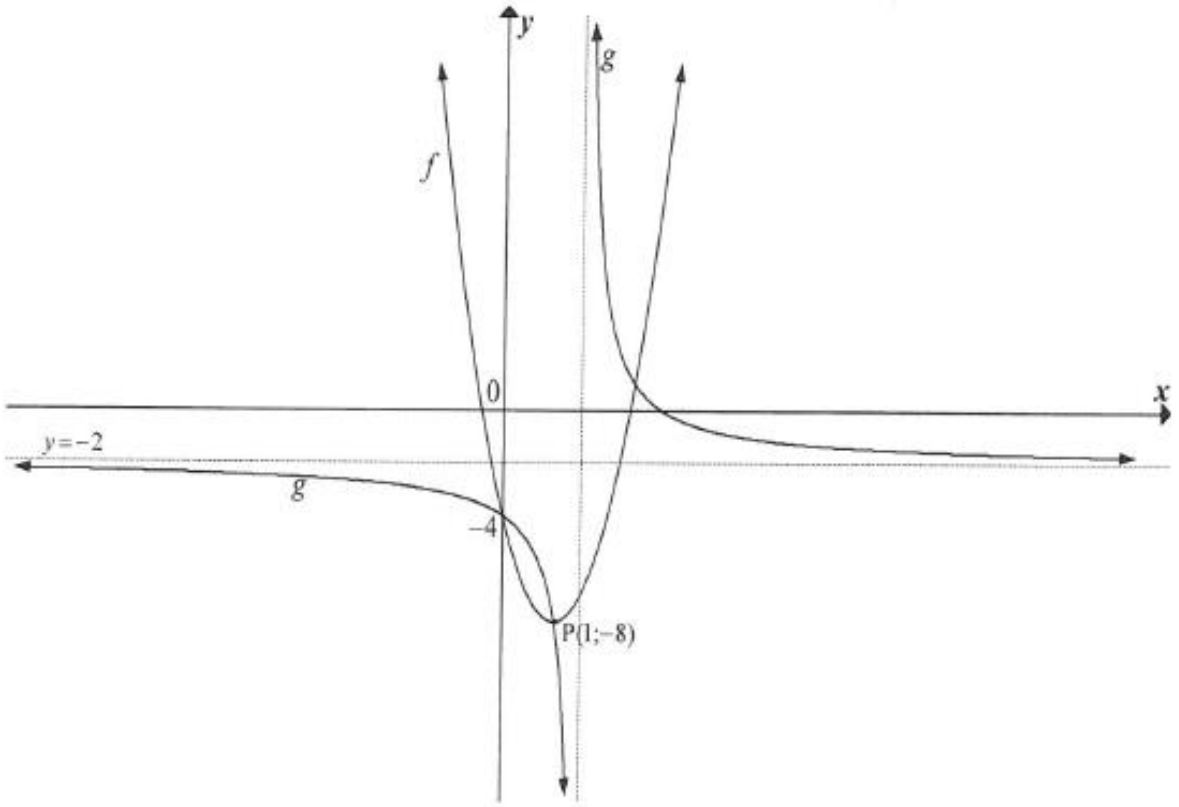
4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

4.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3)

QUESTION 5

The graphs of the functions $f(x)=a(x+p)^2+q$ and $g(x)=\frac{k}{x+r}+d$ are sketched below.

Both graphs cut the y-axis at -4. One of the points of intersection of the graphs is P(1 ; –8), which is also the turning point of f. The horizontal asymptote of g is $y=-2$.



5.1 Calculate the values of a, p and q. (4)

5.2 Calculate the values of k, r and d. (6)

QUESTION 6

Given: $f(x)=\frac{1}{4}x^2, x\le 0$

6.1 Determine the equation of f^{-1} in the form $f^{-1}(x)=\ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

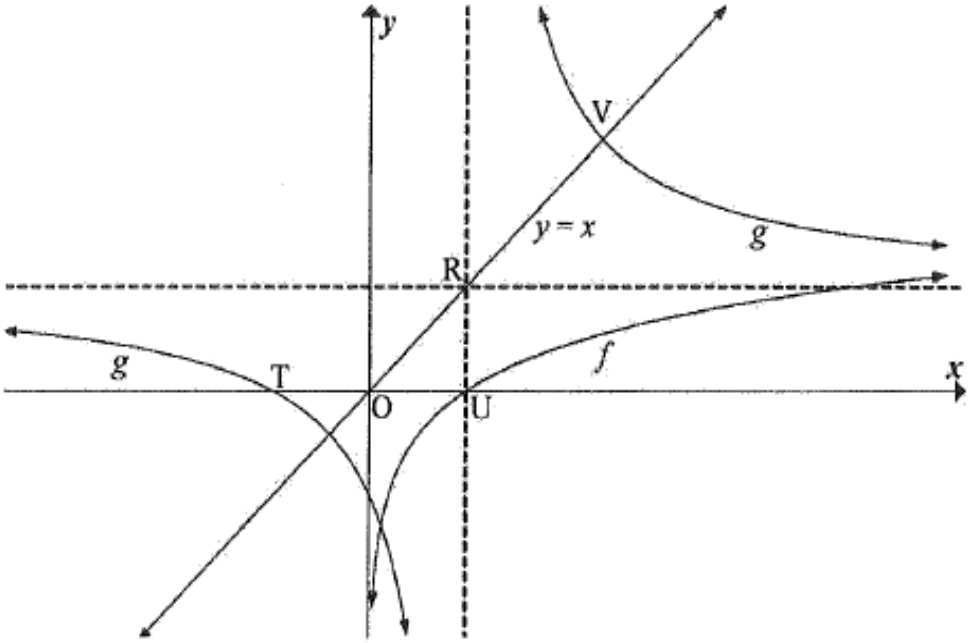
6.3 Is f^{-1} a function? Give a reason for your answer. (2)

[8]

QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the x-intercepts of g and f respectively.
- The line $y = x$ intersects the asymptotes of g at R, and the graph of g at V.



4.1 Write down the coordinates of U. (1)

4.2 Write down the equations of the asymptotes of g. (2)

4.3 Determine the coordinates of T. (2)

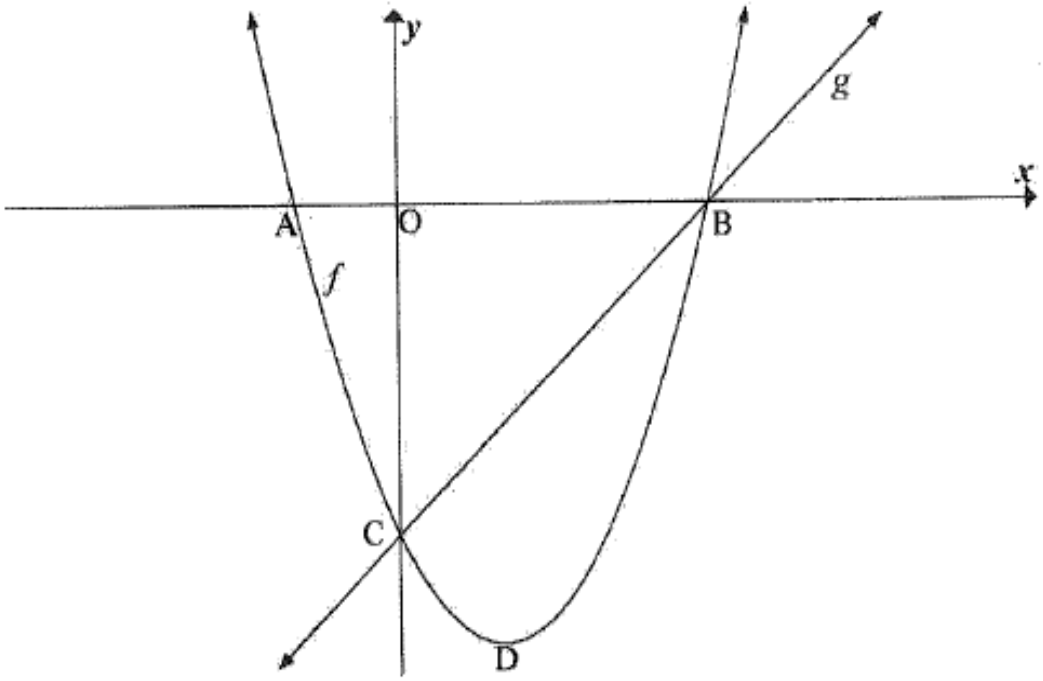
4.4 Write down the equation of h, the reflection of f in the line $y = x$, in the form $y = ...$ (2)

QUESTION 5

5.1 The sketch below shows the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x - 3$.

- A and B are the x-intercepts of f.
- The graphs of f and g intersect at C and B.

D is the turning point of f.



5.1.1 Determine the coordinates of C. (1)

5.1.2 Calculate the length of AB. (4)

5.1.3 Determine the coordinates of D. (2)

5.1.4 Calculate the average gradient of f between C and D. (2)

CALCULUS (BASICS)

NOVEMBER 2008

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (5)

8.2 Determine, using the rules of differentiation:

$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

Show ALL calculations. (3)
[8]

FEB/MARCH 2009

QUESTION 11

11.1 Differentiate f by first principles where $f(x) = x^2 - 2x$. (5)

11.2 Evaluate:

11.2.1 $D_x[(x^3 - 3)^2]$ (3)

11.2.2 $\frac{dy}{dx}$ if $y = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ (3)
[11]

NOVEMBER 2009

QUESTION 10

10.1 Differentiate $f(x)$ from first principles if $f(x) = -2x^2 + 3$. (5)

10.2 Evaluate: $\frac{dy}{dx}$ if $y = x^2 - \frac{1}{2x^3}$ (2)
[7]

FEB/MARCH 2010

QUESTION 10

10.1 Differentiate f from first principles: $f(x)=\frac{1}{x}$ (4)

10.2 Use the rules of differentiation to determine $\frac{dy}{dx}$ if $y=(2-5x)^2$ (3)

[7]

NOVEMBER 2010

QUESTION 8

8.1 Differentiate $g(x)=x^2-5$ from first principles. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y=\frac{x^6}{2}+4\sqrt{x}$. (3)

FEB/MARCH 2011

QUESTION 9

9.1 Use the definition to differentiate $f(x)=1-3x^2$. (Use first principles.) (4)

9.2 Calculate $D_x\left[4-\frac{4}{x^3}-\frac{1}{x^4}\right]$. (3)

9.3 Determine $\frac{dy}{dx}$ if $y=\left(1+\sqrt{x}\right)^2$. (3)

[10]

NOVEMBER 2011

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=-4x^2$. (5)

8.2 Evaluate:

8.2.1 $\frac{dy}{dx}$ if $y=\frac{3}{2x}-\frac{x^2}{2}$ (3)

8.2.2 $f'(1)$ if $f(x)=(7x+1)^2$ (4)

[12]

FEB/MARCH 2012

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$. (5)

8.2 Evaluate:

8.2.1 $D_x[1 + 6\sqrt{x}]$ (2)

8.2.2 $\frac{dy}{dx}$ if $y = \frac{8 - 3x^6}{8x^5}$ (4)

[11]

NOVEMBER 2012

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 5$. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = x^{-4} + 2x^3 - \frac{x}{5}$. (3)

8.3 Given: $g(x) = \frac{x^2 + x - 2}{x - 1}$

8.3.1 Calculate $g'(x)$ for $x \neq 1$. (2)

MARCH 2013

QUESTION 9

9.1 Use the definition of the derivative (first principles) to determine $f'(x)$ if $f(x) = 2x^3$ (5)

9.2 Determine $\frac{dy}{dx}$ if $y = \frac{2\sqrt{x} + 1}{x^2}$ (4)

MARCH 2015

QUESTION 8

8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

MARCH 2017

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

NOVEMBER 2009

QUESTION 11

Given: $f(x) = -x^3 + x^2 + 8x - 12$

11.1 Calculate the x-intercepts of the graph of f. (5)

11.2 Calculate the coordinates of the turning points of the graph of f. (5)

11.3 Sketch the graph of f, showing clearly all the intercepts with the axes and turning points. (3)

11.4 Write down the x-coordinate of the point of inflection of f. (2)

11.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)

[17]

FEB/MARCH 2011

QUESTION 10

Given: $g(x) = (x-6)(x-3)(x+2)$

10.1 Calculate the y-intercept of g. (1)

10.2 Write down the x-intercepts of g. (2)

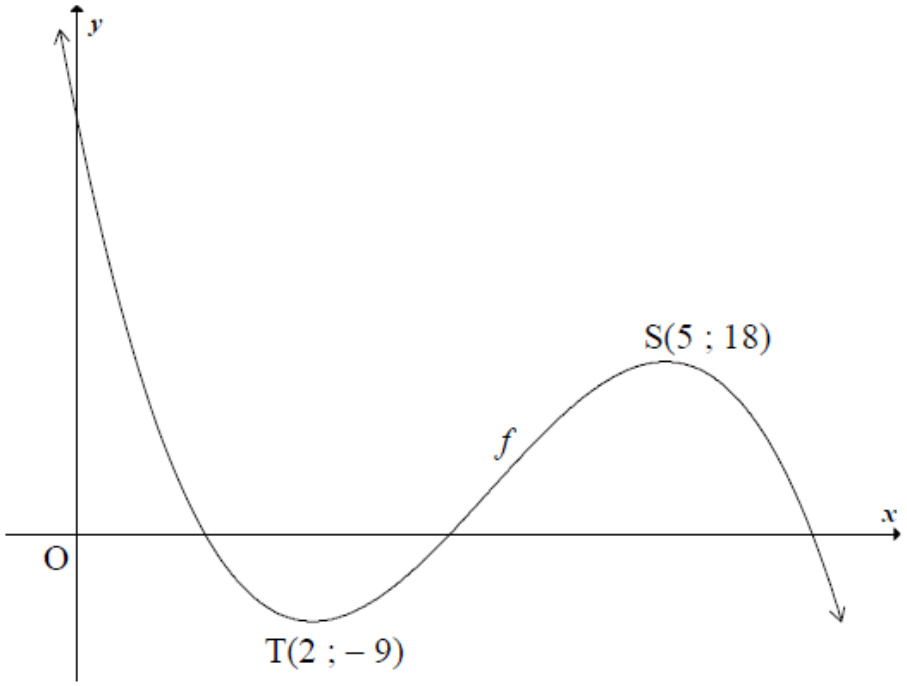
10.3 Determine the turning points of g. (6)

10.4 Sketch the graph of g on DIAGRAM SHEET 2. (4)

NOVEMBER 2011

QUESTION 9

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.
The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).



9.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)

9.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

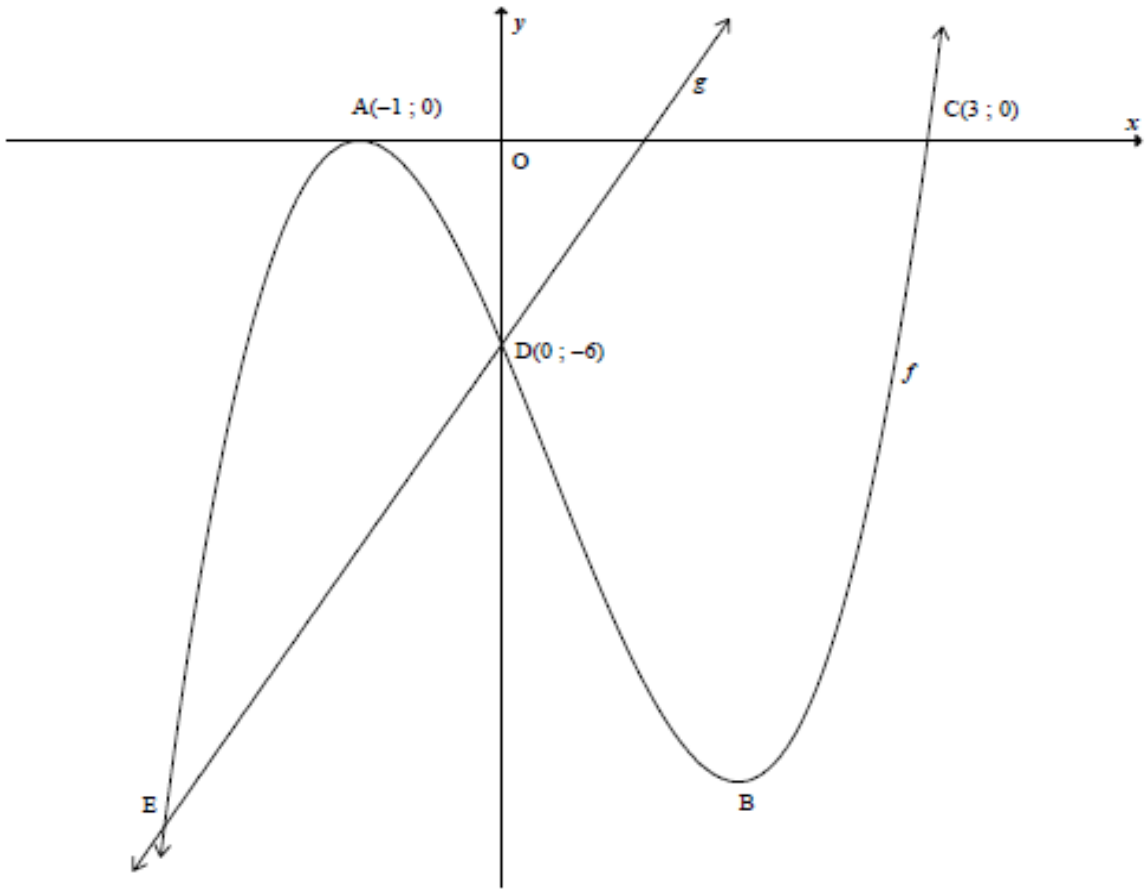
9.3 Determine the x-value at which the graph of f has a point of inflection. (2)

[14]

FEBRUARY MARCH 2012

QUESTION 9

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A(– 1 ; 0) and C(3 ; 0) are the x-intercepts of f.
The graph of f has turning points at A and B.
D(0 ; – 6) is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.



9.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

9.2 Calculate the coordinates of the turning point B. (5)

MARCH 2013

QUESTION 10

Given: $f(x) = -x^3 - x^2 + x + 10$

10.1 Write down the coordinates of the y-intercept of f. (1)

10.2 Show that (2 ; 0) is the only x-intercept of f. (4)

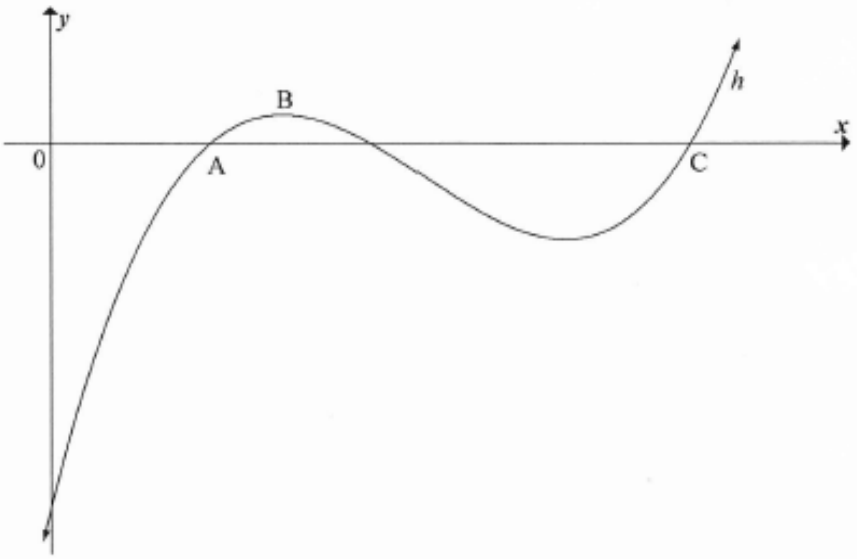
10.3 Calculate the coordinates of the turning points of f. (6)

10.4 Sketch the graph of f in your ANSWER BOOK. Show all intercepts with the axes and all turning points. (3)

[14]

MARCH 2015

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

8.3.3 Calculate the coordinates of C. (4)

MARCH 2016

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x-2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

NOVEMBER 2016

QUESTION 9

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

MARCH 2017

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

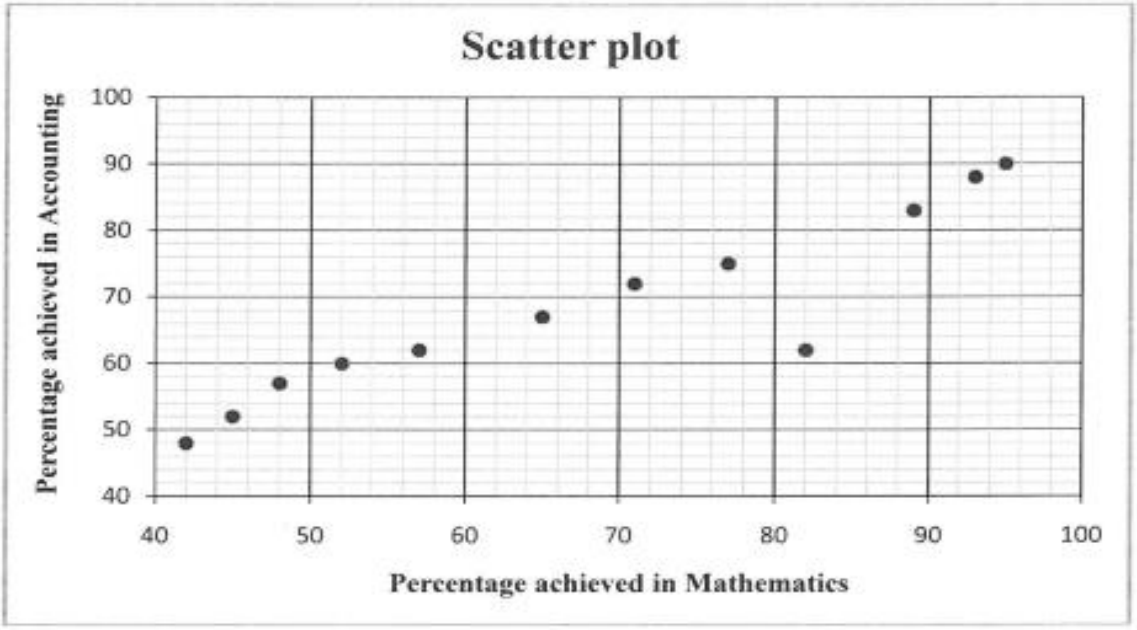
PAPER 2

STATISTICS

QUESTION 1

At a certain school, only 12 candidates take Mathematics and Accounting. The marks, as a percentage, scored by these candidates in the preparatory examinations for Mathematics and Accounting, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62



1.1 Calculate the mean percentage of the Mathematics data. (2)

1.2 Calculate the standard deviation of the Mathematics data. (1)

1.3 Determine the number of candidates whose percentages in Mathematics lie within ONE standard deviation of the mean. (3)

1.4 Calculate an equation for the least squares regression line (line of best fit) for the data. (3)

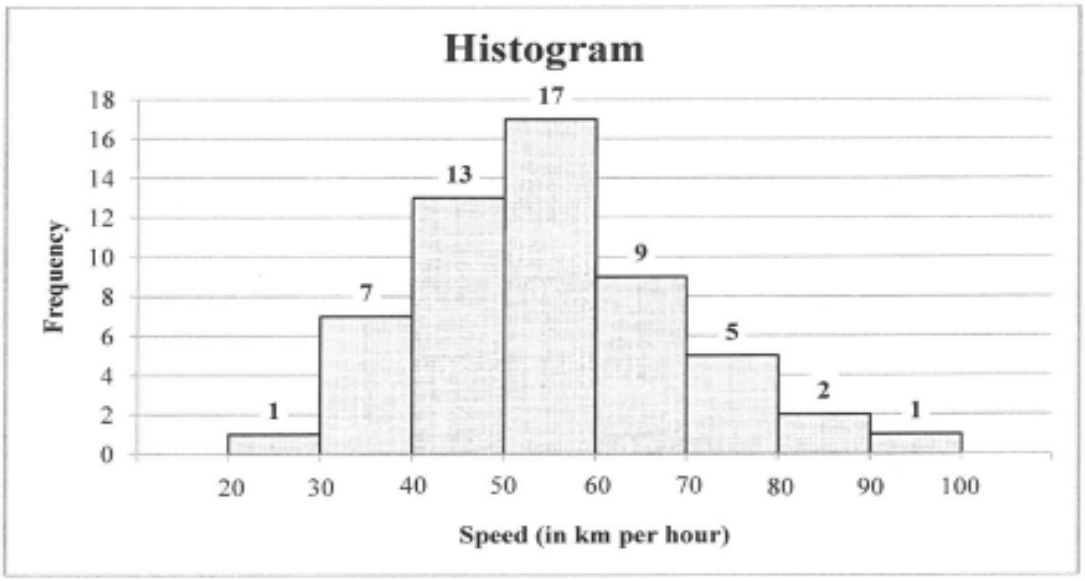
1.5 If a candidate from this group scored 60% in the Mathematics examination but was absent for the Accounting examination, predict the percentage that this candidate would have scored in the Accounting examination, using your equation in QUESTION 1.4. (Round off your answer to the NEAREST INTEGER.) (2)

1.6 Use the scatter plot and identify any outlier(s) in the data. (1)

[12]

QUESTION 2

The speeds of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.



2.1 Identify the modal class of the data. (1)

2.2 Use the histogram to:

2.2.1 Complete the cumulative frequency column in the table on DIAGRAM SHEET 1 (2)

2.2.2 Draw an ogive (cumulative frequency graph) of the above data on the grid on DIAGRAM SHEET 1 (3)

2.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorists who will receive a speeding fine. (2)
[8]

Feb March 2015

QUESTION 1

The table below shows the distances (in kilometres) travelled daily by a sales representative for 21 working days in a certain month.

131	132	140	140	141	144	146
147	149	150	151	159	167	169
169	172	174	175	178	187	189

1.1 Calculate the mean distance travelled by the sales representative. (2)

1.2 Write down the five-number summary for this set of data. (4)

1.3 Use the scaled line on DIAGRAM SHEET 1 to draw a box-and-whisker diagram for this set of data. (2)

1.4 Comment on the skewness of the data. (1)

1.5 Calculate the standard deviation of the distance travelled. (2)

1.6 The sales representative discovered that his odometer was faulty. The actual reading on each of the 21 days was p km more than that which was indicated. Write down, in terms of p (if applicable), the:

1.6.1 Actual mean (1)

1.6.2 Actual standard deviation (1)

[13]

QUESTION 1

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK. (3)

1.2 The equation of the least squares regression line is $\hat{y} = 154{,}60 + 77{,}13x$.

1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100 kJ. (2)

1.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1. (2)

1.3 Identify an outlier in the data set. (1)

1.4 Calculate the value of the correlation coefficient. (2)

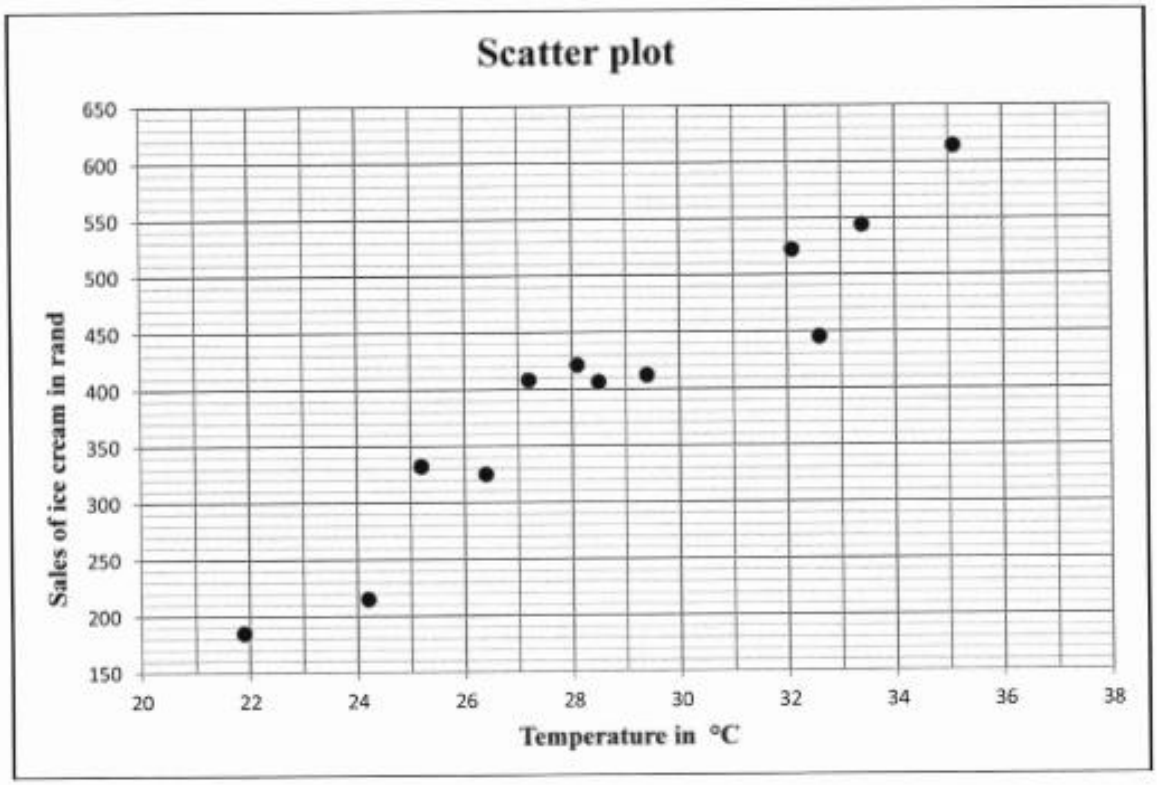
1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1)

[11]

QUESTION 2

An ice-cream shop recorded the sales of ice cream, in rand, and the maximum temperature, in °C, for 12 days in a certain month. The data that they collected is represented in the table and scatter plot below.

Temperature in °C	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Sales of ice cream in rand	215	325	185	332	406	522	412	614	544	421	445	408



2.1 Describe the influence of temperature on the sales of ice cream in the scatter plot. (1)

2.2 Give a reason why this trend cannot continue indefinitely. (1)

2.3 Calculate an equation for the least squares regression line (line of best fit). (4)

2.4 Calculate the correlation coefficient. (1)

2.5 Comment on the strength of the relationship between the variables. (1)

[8]

QUESTION 2

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

2.3 Determine the standard deviation of the data. (2)

2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)
[9]

QUESTION 2

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

2.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places. (3)

2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (4)

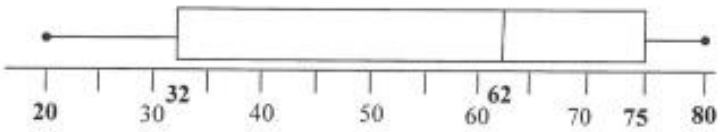
2.3 Hence, estimate the number of days on which 65 or more messages were sent. (2)

[9]

Feb/March 2016

QUESTION 1

The box and whisker diagram below shows the marks (out of 80) obtained in a History test by a class of nine learners.



1.1 Comment on the skewness of the data. (1)

1.2 Write down the range of the marks obtained. (2)

1.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (2)

1.4 In ascending order, the second mark is 28, the third mark 36 and the sixth mark 69. The seventh and eighth marks are the same. The average mark for this test is 54.

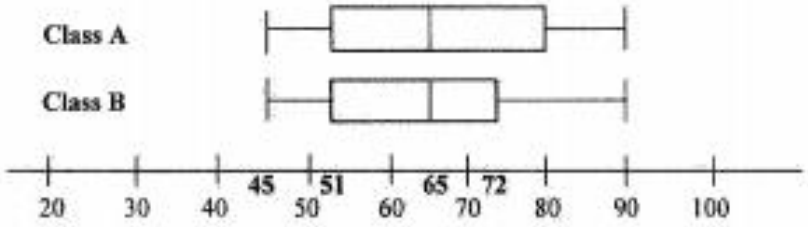
	28	36			69			

Fill in the marks of the remaining learners in ascending order. (6)

[11]

QUESTION 2

2.1 Mrs Smith has two classes, each having 30 learners. Their final marks (out of 100) for the year are represented in the box and whisker diagram below.



2.1.1 Determine the interquartile range of Class B. (2)

2.1.2 Explain the significance in the difference of the length of the boxes in the diagram. (2)

2.1.3 Mrs Smith studied the results and made the comment that there was no significant difference in the performance of the two classes. Give TWO reasons you think Mrs Smith will use to prove her statement. (2)

2.2 Eight couples entered a dance competition. Their performances were scored by two judges. The scores (out of 20) are given in the table below.

COUPLE	1	2	3	4	5	6	7	8
JUDGE 1	18	4	6	8	5	12	10	14
JUDGE 2	15	6	3	5	5	14	8	15

2.2.1 Determine the equation of the least squares regression line of the scores given by the two judges. (3)

2.2.2 A ninth couple entered late for the competition and received a score of 15 from JUDGE 1. Estimate the score that might have been assigned by JUDGE 2 to the nearest integral value. (2)

2.2.3 Are the judges consistent in assigning scores to the performance of the couples? Prove your answer and support it with relevant statistics. (2)

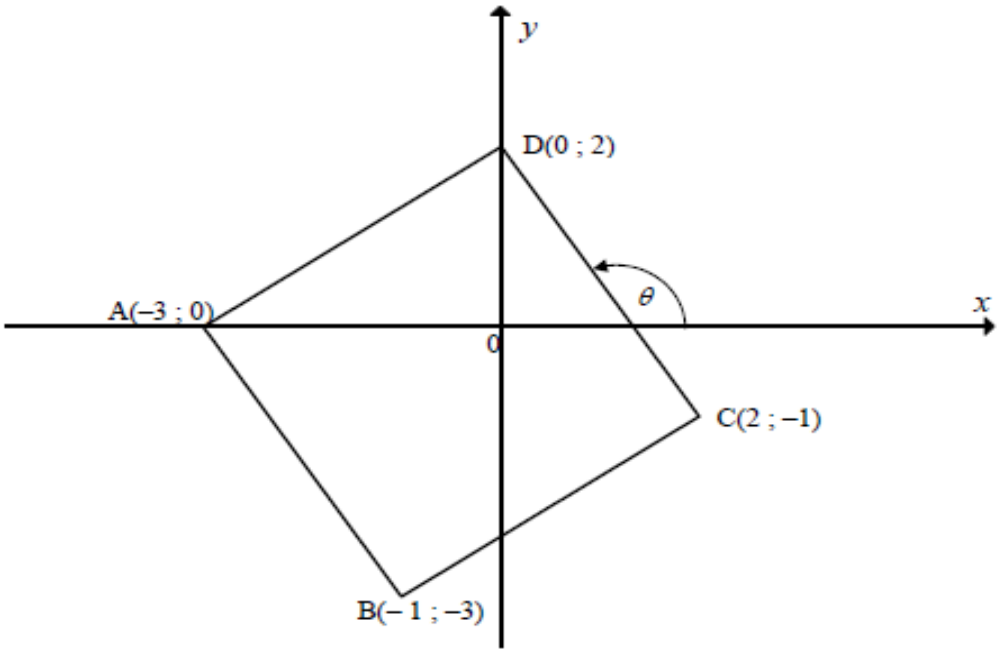
[13]

ANALYTICAL GEOMETRY BASICS

NOVEMBER 2008

QUESTION 1

ABCD is a quadrilateral with vertices A(– 3 ; 0), B(– 1 ; – 3), C(2 ; –1) and D(0 ; 2).



1.1 Determine the coordinates of M, the midpoint of AC. (2)

1.2 Show that AC and BD bisect each other. (3)

1.3 Prove that $A\hat{D}C = 90°$. (4)

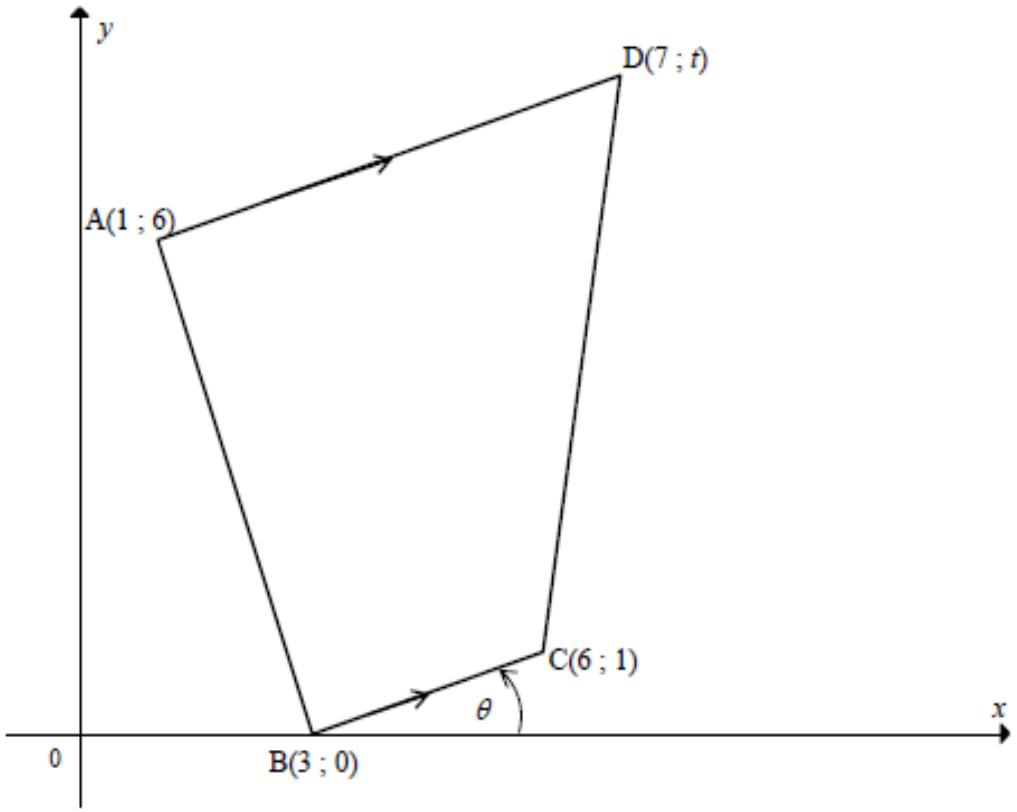
1.4 Show that ABCD is a square. (6)

1.5 Determine the size of θ, the angle of inclination of DC, correct to ONE decimal place. (3)

FEB/MARCH 2009

QUESTION 1

ABCD is a quadrilateral with vertices A(1 ; 6), B(3 ; 0), C(6 ; 1) and D(7 ; t) in a Cartesian plane. AD ∥ BC.



1.1 Calculate the gradient of BC. (2)

1.2 Determine the equation of AD in the form $y = \ldots$ (3)

1.3 Show that $t = 8$. (2)

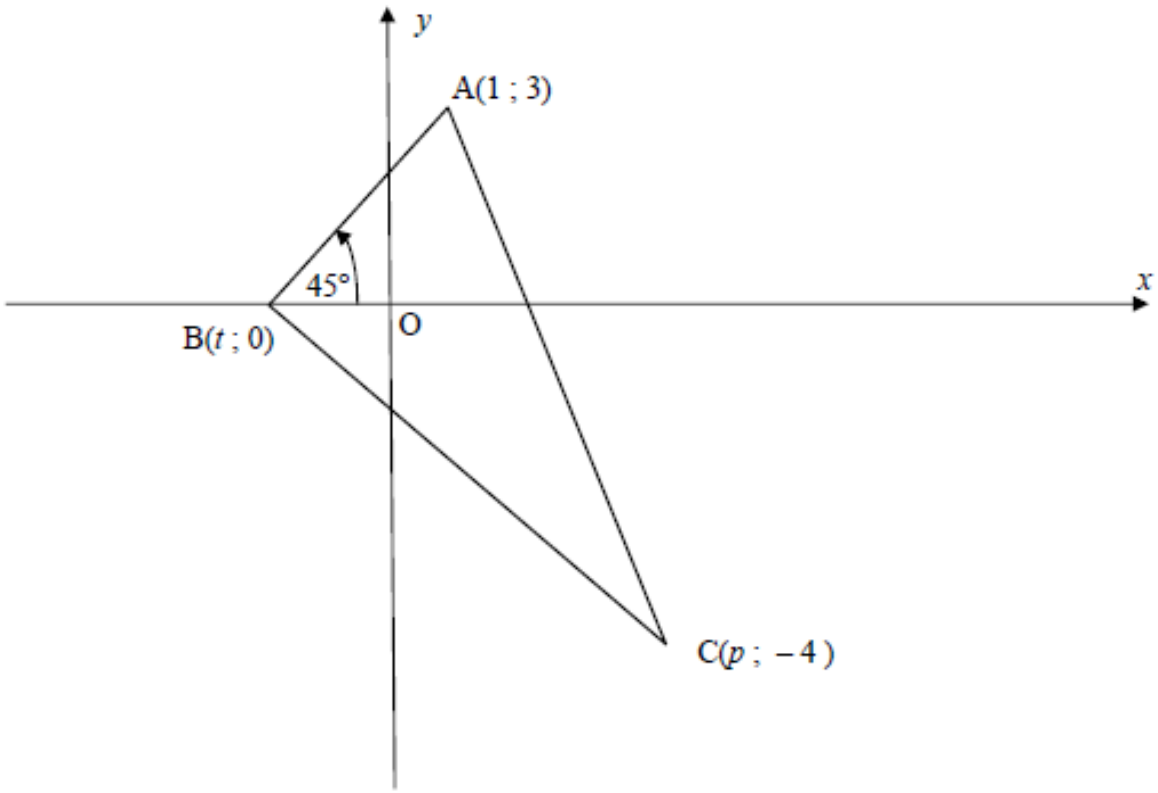
1.4 Calculate the lengths of AD, BC and AB. (4)

1.5 Show that AB is perpendicular to BC. (3)

NOVEMBER 2009

QUESTION 4

ABC is a triangle with vertices A(1 ; 3), B(t ; 0) and C(p ; −4), with $p > 0$, in a Cartesian plane.
AB makes an angle of 45° with the positive x-axis. $AC = \sqrt{50}$.



4.1 Determine the gradient of AB. (2)

4.2 Calculate the value of t. (2)

4.3 Calculate p, the x-coordinate of point C. (4)

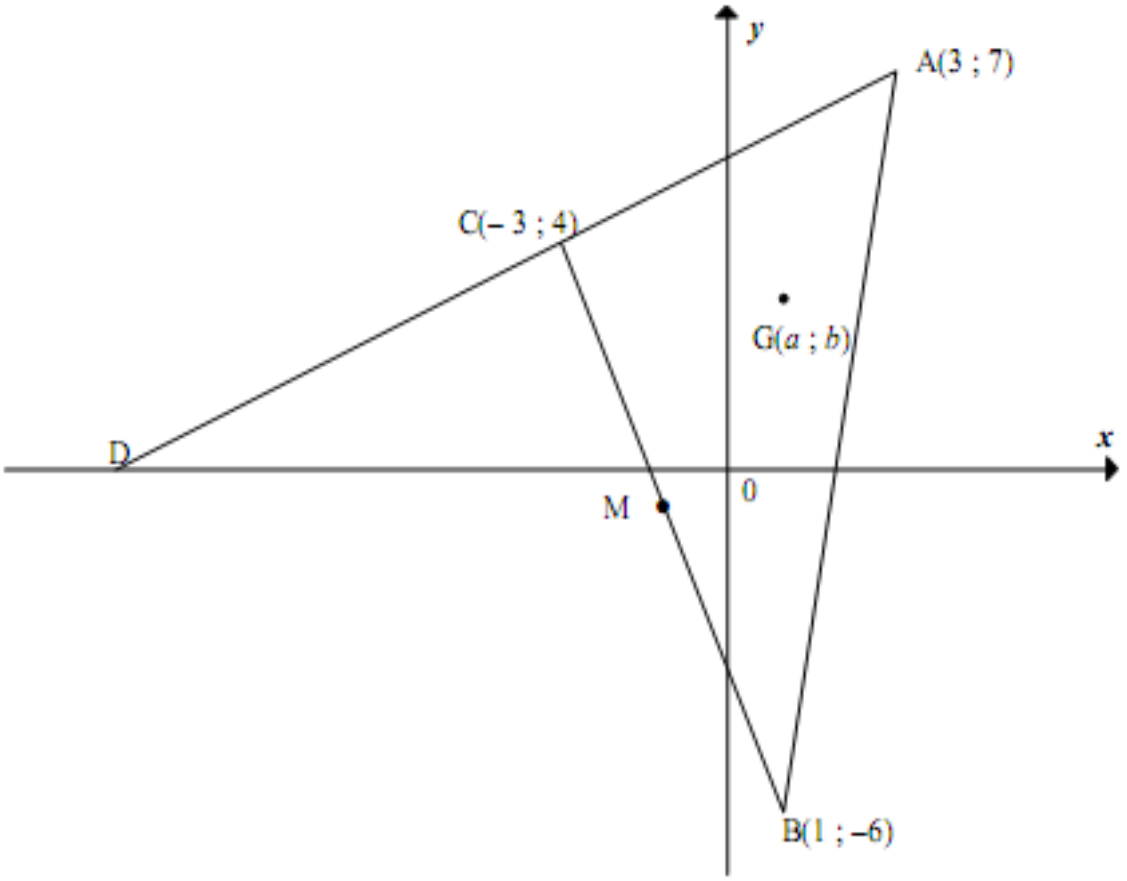
4.4 Hence, determine the midpoint of BC. (2)

4.5 Determine the equation of the line parallel to AB, passing through point C. (3)
[13]

NOVEMBER 2010

QUESTION 5

In the diagram below, A, B and C are the vertices of a triangle. AC is extended to cut the x-axis at D.



5.1 Calculate the gradient of:

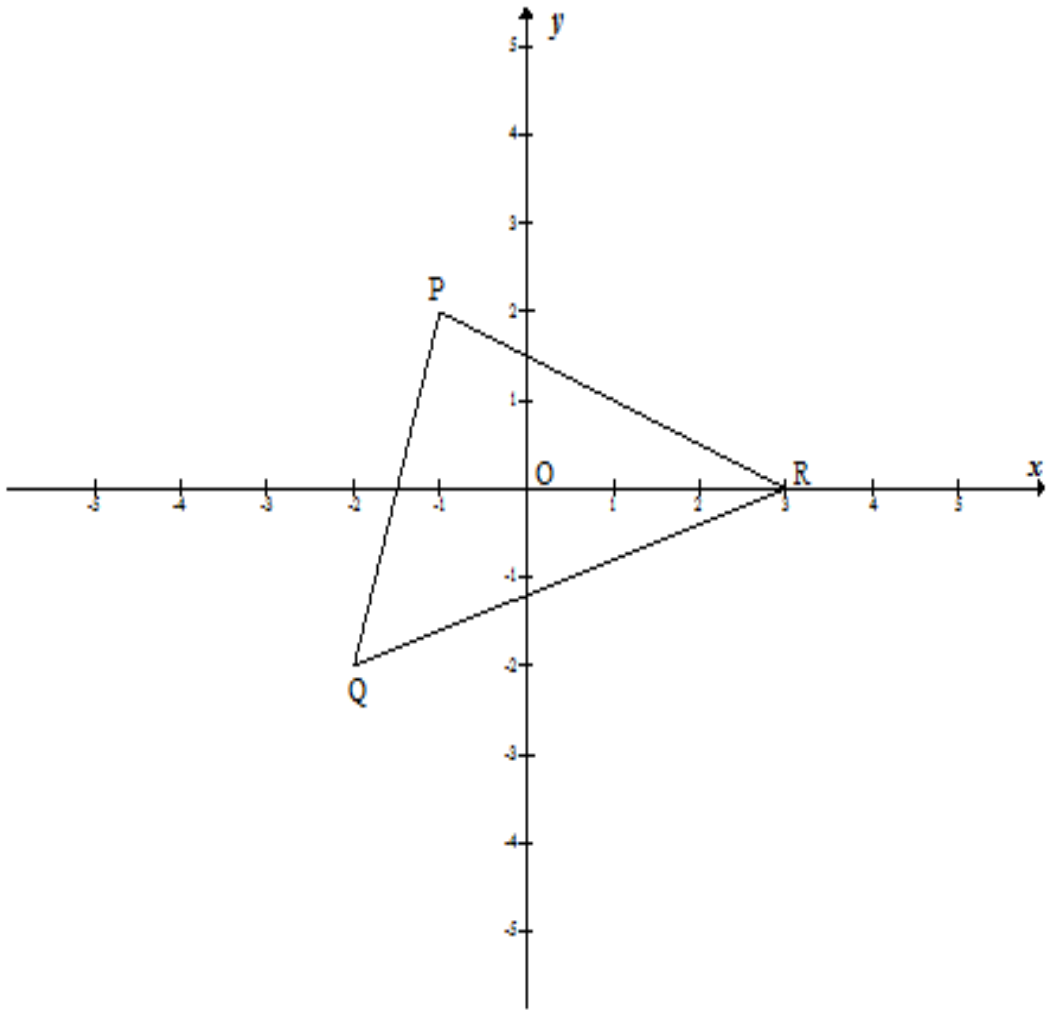
5.1.1 AD (2)

5.1.2 BC (1)

March 2011

QUESTION 4

In the diagram below ΔPQR with vertices P(– 1 ; 2), Q(– 2 ; – 2) and R(3 ; 0) is given.



4.1 Calculate the angle that PQ makes with the positive x-axis. (3)

4.2 Determine the coordinates of M, the midpoint of PR. (2)

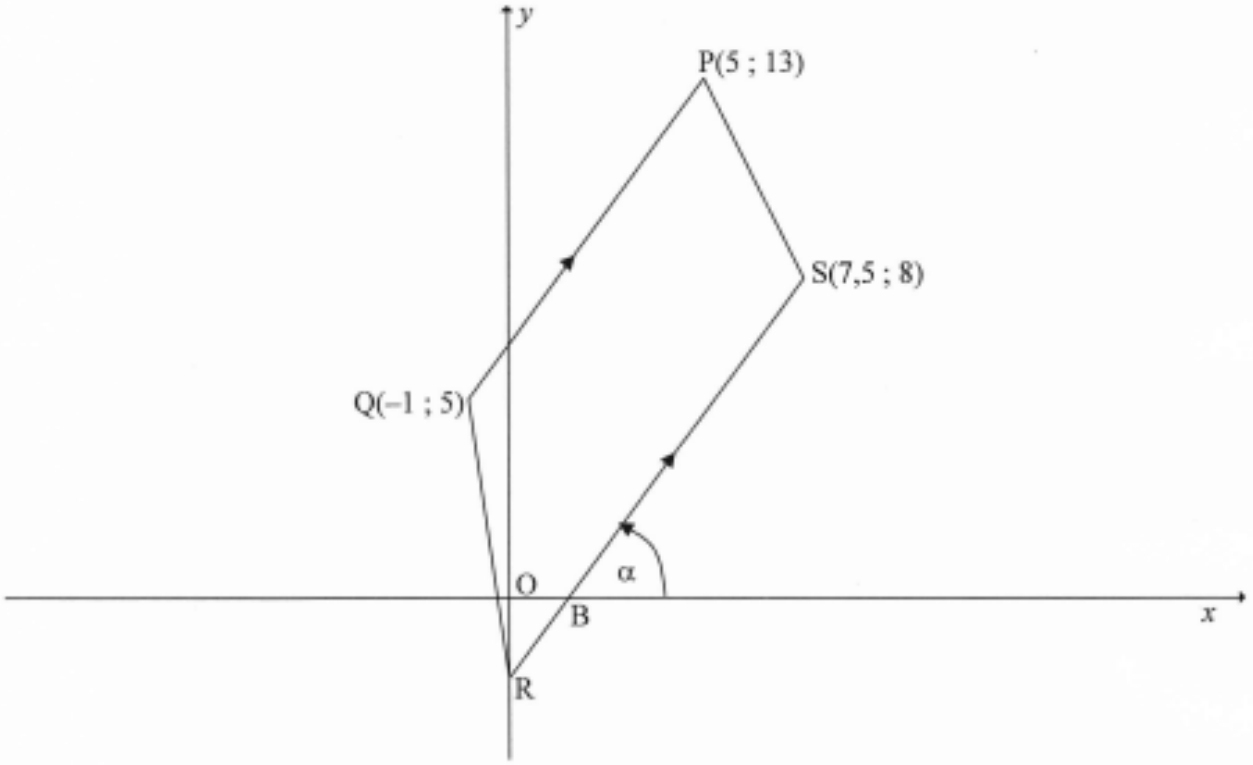
4.3 Determine the perimeter of ΔPQR to the nearest whole number. (5)

4.4 Determine an equation of the line parallel to PQ that passes through M. (3)

[13]

QUESTION 3

In the diagram below points P(5 ; 13), Q(–1 ; 5) and S(7,5 ; 8) are given. SR | | PQ where R is the y-intercept of SR. The x-intercept of SR is B. QR is joined.



3.1 Calculate the length of PQ. (3)

3.2 Calculate the gradient of PQ. (2)

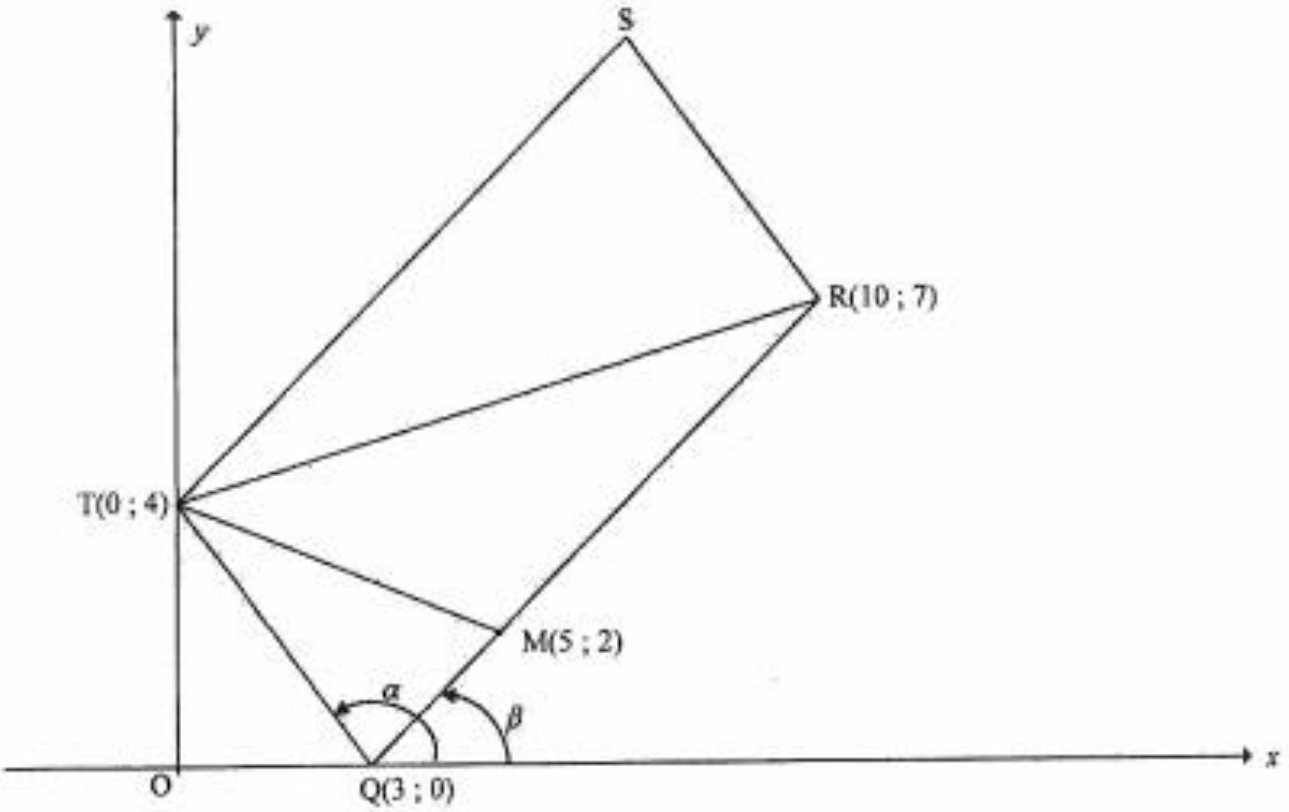
3.3 Determine the equation of line RS in the form $ax + by + c = 0$. (4)

3.4 Determine the x-coordinate of B. (2)

March 2017

QUESTION 3

In the diagram, Q(3 ; 0), R(10 ; 7), S and T(0 ; 4) are the vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M(5 ; 2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ. (1)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

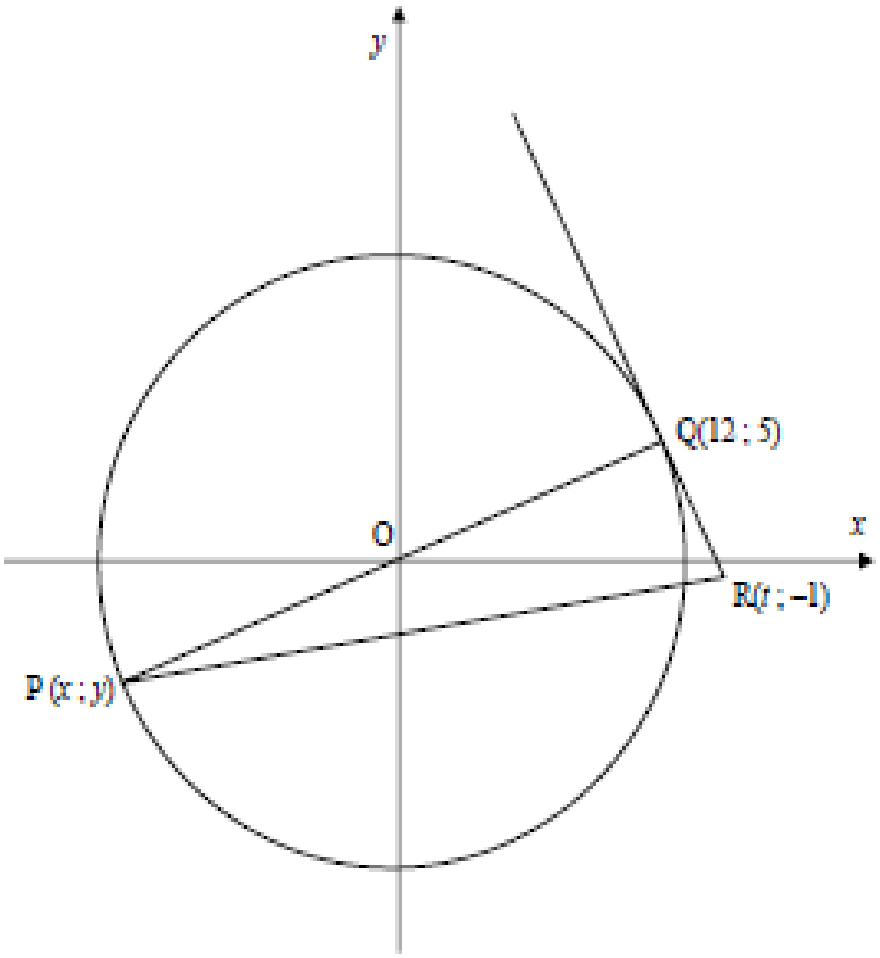
3.3 F(k ; −8) is a point in the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

ANALYTICAL GEOMETRY (CIRCLES)

NOVEMBER 2008

QUESTION 2

O is the centre of the circle in the figure below. P(x ; y) and Q (12 ; 5) are two points on the circle. POQ is a straight line. The point R(t ; – 1) lies on the tangent to the circle at Q.

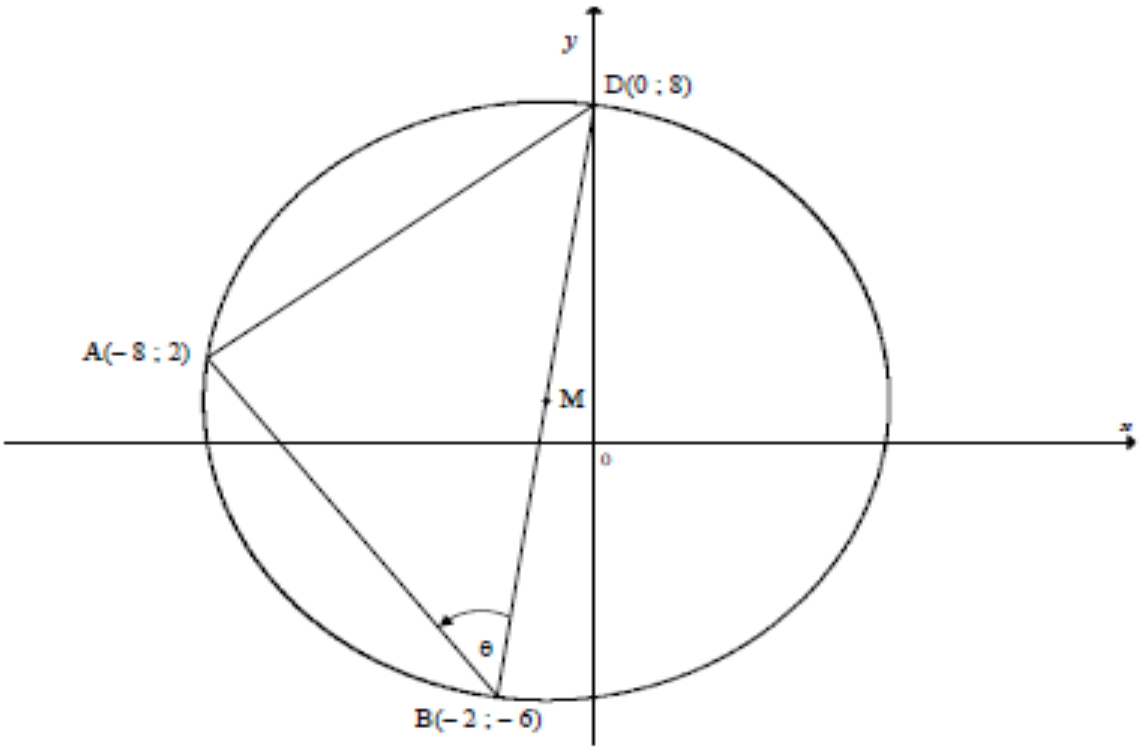


2.1	Determine the equation of the circle.	(3)
2.2	Determine the equation of the straight line through P and Q.	(2)
2.3	Determine x and y, the coordinates of P.	(2)
2.4	Show that the gradient of QR is $-\frac{12}{5}$.	(2)
2.5	Determine the equation of the tangent QR in the form $y =$ ...	(3)
2.6	Calculate the value of t.	(2)
2.7	Determine an equation of the circle with centre Q(12 ; 5) and passing through the origin.	(3)
		[17]

NOVEMBER 2009

QUESTION 5

A(−8 ; 2), B(−2 ; −6) and D(0 ; 8) are the vertices of a triangle that lies on the circumference of a circle with diameter BD and centre M, as shown in the figure below.



5.1 Calculate the coordinates of M. (2)

5.2 Show that (–8 ; 2) lies on the line $y = 7x + 58$. (1)

5.3 What is the relationship between the line $y = 7x + 58$ and the circle centred at M? Motivate your answer. (5)

5.4 Calculate the lengths of AD and AB. (4)

5.5 Prove $D\hat{A}B = 90°$. (3)

5.6 Write down the size of angle θ. (1)

QUESTION 6

6.1 Determine the centre and radius of the circle with the equation $x^2 + y^2 + 8x + 4y - 38 = 0$. (4)

6.2 A second circle has the equation $(x-4)^2 + (y-6)^2 = 26$. Calculate the distance between the centres of the two circles. (2)

FEB/MARCH 2011

QUESTION 5

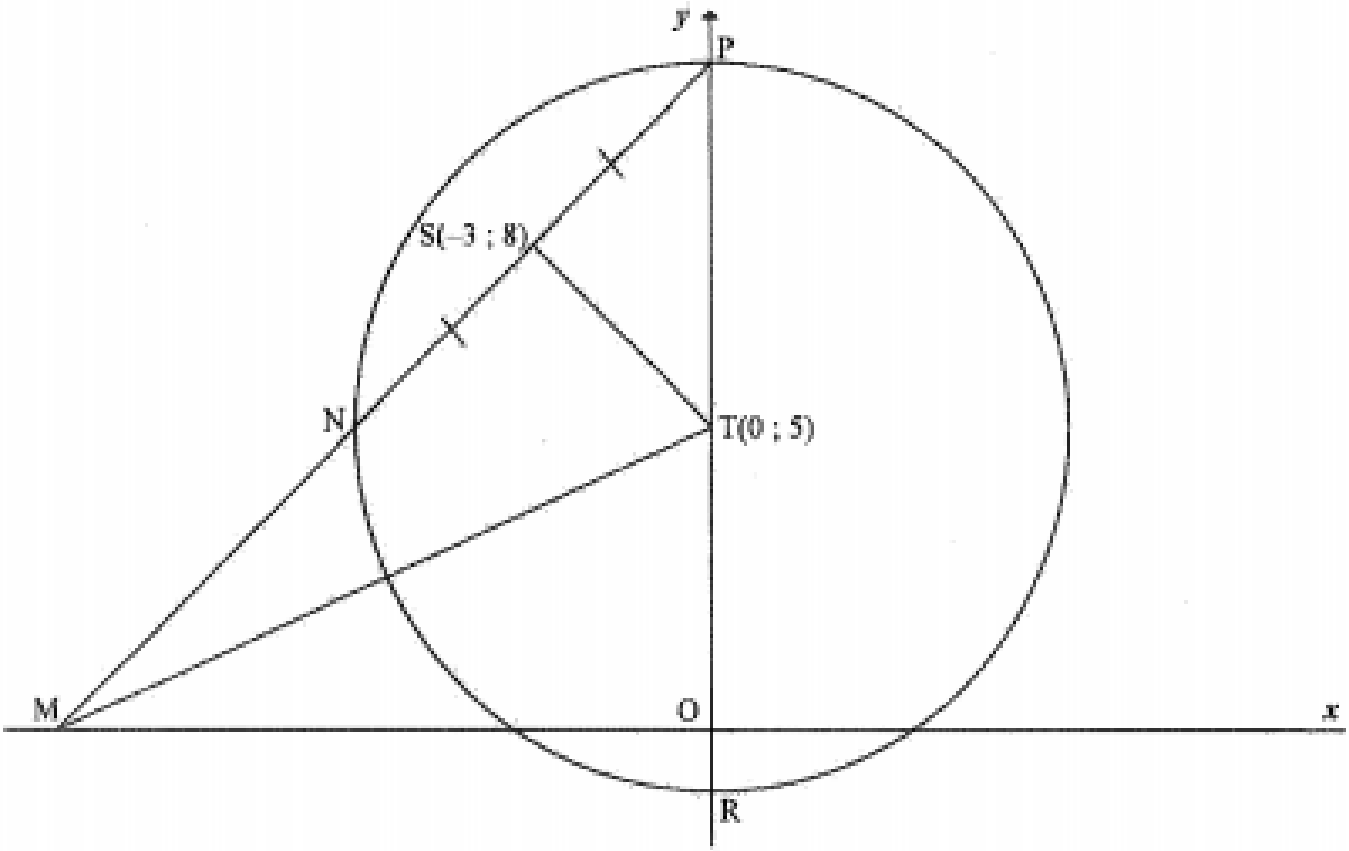
5.1 The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.

5.1.1 Prove that the point $(2 ; -9)$ is on the circumference of the circle. (2)

5.1.2 Determine an equation of the tangent to the circle at the point $(2 ; -9)$. (7)

QUESTION 4

In the diagram, the circle, having centre T(0 ; 5), cuts the y-axis at P and R. The line through P and S(–3 ; 8) intersects the circle at N and the x-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line passing through N and P in the form $y = mx + c$. (5)

4.3 Determine the equations of the tangents to the circle that are parallel to the x-axis. (4)

4.4 Determine the length of MT. (4)