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NATIONAL CURRICULUM STATEMENT GRADES 10-12 (GENERAL)

LEARNING PROGRAMME GUIDELINES

MATHEMATICS

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CONTENTS

SECTION 1

INTRODUCTION

1.1 INTRODUCING THE NATIONAL CURRICULUM STATEMENT

1.1.1 BACKGROUND

In 1995 the South African government began the process of developing a new curriculum for the school system. There were two imperatives for this. First, the scale of change in the world, the growth and development of knowledge and technology and the demands of the 21st Century required learners to be exposed to different and higher level skills and knowledge than those required by the existing South African curricula. Second, South Africa had changed. The curricula for schools therefore required revision to reflect new values and principles, especially those of the Constitution of South Africa.

The first version of the new curriculum for the General Education Band, known as Curriculum 2005, was introduced into the Foundation Phase in 1997. While there was much to commend the curriculum, the concerns of teachers led to a review of the Curriculum in 1999. The review of Curriculum 2005 provides the basis for the development of the National Curriculum Statement for General Education and Training (Grades R-9) and the National Curriculum Statement for Grades 10-12.

1.1.2 THE NATIONAL CURRICULUM STATEMENT

The National Curriculum Statement consists of 29 subjects. Subject specialists developed the Subject Statements which make up the National Curriculum Statement. The draft versions of the Subject Statements were published for comment in 2001 and then re-worked to take account of the comments received. In 2002 twenty-four subject statements and an overview document were declared policy through Government Gazette. In 2004 five subjects were added to the National Curriculum Statement. The National Curriculum Statement now consists of the Subject Statements for the following subjects:

- Languages – 11 official languages (each counted as three subjects to cater for the three levels Home Language, First Additional Language and Second Additional Language); 13 non-official languages
- Mathematics; Mathematical Literacy; Physical Sciences; Life Sciences; Computer Applications Technology; Information Technology
- Accounting; Business Studies; Economics
- Geography; History; Life Orientation; Religion Studies
- Consumer Studies; Hospitality Studies; Tourism
- Dramatic Arts; Dance Studies; Design; Music; Visual Arts
- Agricultural Sciences, Agricultural Management Practices, Agricultural Technology

- Civil Technology; Mechanical Technology; Electrical Technology; Engineering Graphics and Design

1.1.3 NATIONAL SENIOR CERTIFICATE

The *National Senior Certificate: A Qualification on Level 4 of the National Qualifications Framework (NQF)* provides the requirements for promotion at the end of Grades 10 and 11 and the awarding of the National Senior Certificate at the end of Grade 12. This document replaces two of the original National Curriculum Statement documents: the *Overview* and the *Qualifications and Assessment Policy Framework.*

1.1.4 SUBJECT ASSESSMENT GUIDELINES

The Subject Assessment Guidelines set out the internal or school-based assessment requirements for each subject and the external assessment requirements. In addition, the *National Protocol for Recording and Reporting (Grades R-12)* (an addendum to the policy, *The National Senior Certificate*) has been developed to standardise the recording and reporting procedures for Grades R to 12. This protocol came into effect on 1 January 2007.

1.2 INTRODUCING THE LEARNING PROGRAMME GUIDELINES

1.2.1 PURPOSE AND CONTENT OF THE LEARNING PROGRAMME GUIDELINES

The Learning Programme Guidelines aim to assist teachers and schools in their planning for the introduction of the National Curriculum Statement. The Learning Programme Guidelines should be read in conjunction with the National Senior Certificate policy and the National Curriculum Statement Subject Statements.

Section 2 of the Learning Programme Guidelines suggests how teaching the particular subject may be informed by the principles which underpin the National Curriculum Statement.

Section 3 suggests how schools and teachers might plan for the introduction of the National Curriculum Statement. The Department of Education encourages careful planning to ensure that the high skills, high knowledge goals of the National Curriculum Statement are attained.

The Learning Programme Guidelines do not include sections on assessment. The assessment requirements for each subject are provided in the Subject Assessment Guidelines which come into effect on 1 January 2008.

1.2.2 WHAT IS A LEARNING PROGRAMME

INTRODUCTION

A Learning Programme assists teachers to plan for sequenced learning, teaching and assessment in Grades 10 to 12 so that all Learning Outcomes in a subject are achieved in a progressive manner. The following three phases of planning are recommended:

11 Phase 1 – develop a *Subject Framework* for grades 10 to 12

12 Phase 2 – develop a *Work Schedule* for each grade

13 Phase 3 – develop *Lesson Plans*

It is recommended that the teachers of a subject at a school or cluster of schools first put together a broad subject outline (Subject Framework) for the three grades to arrive at an understanding of the content of the subject and the progression which needs to take place across the grades (see Section 3.3.1). This will assist with the demarcation of content for each grade. Thereafter, teachers teaching the same grade need to work together to develop a year long Work Schedule for their subject. The Work Schedule should indicate the sequence in which the content and context will be presented for the subject in that particular grade (see Section 3.3.2). Finally, individual teachers should design Lesson Plans using the grade-specific Work Schedule as the starting point. The Lesson Plans should include learning, teaching and assessment activities that reflect the Learning Outcomes and Assessment Standards set out in the Subject Statements (see Section 3.3.3). Learning Programmes should accommodate diversity in schools and classrooms but reflect the core content of the national curriculum.

An outline of the process involved in the design of a Learning Programme is provided on page 6.

DESIGNING A LEARNING PROGRAMME

A detailed description of the process involved in the design of a Learning Programme is provided in Sections 3.3.1 – 3.3.3 of the Learning Programme Guidelines. The first stage, the development of a Subject Framework does not require a written document but teachers are strongly advised to spend time with subject experts in developing a deep understanding of the skills, knowledge and values set out in the Subject Statements. The quality and rigour of this engagement will determine the quality of teaching and learning in the classroom.

Once the Subject Framework has been completed, teachers should develop Work Schedules and Lesson Plans. Examples of Work Schedules and Lesson Plans are provided in the Learning Programme Guidelines. Teachers are encouraged to critically engage with these formats and develop their own.

Developing a Subject Framework (Grades 10-12)

Planning for the teaching of subjects in Grades 10 to 12 should begin with a detailed examination of the scope of the subject as set out in the Subject Statement. No particular format or template is recommended for this first phase of planning but the steps recommended should be used as a checklist.

Although no prescribed document is required for this stage of planning, school-wide planning (timetables, requisitioning, teacher development, classroom allocation) as well as the development of grade-specific work schedules would benefit from short documents which spell out:

- The scope of the subject – the knowledge, skills and values; the content; the contexts or themes; electives etc. to be covered in the three grades for each subject
- A three-year assessment plan for the subject
- The list of LTSM required for the subject

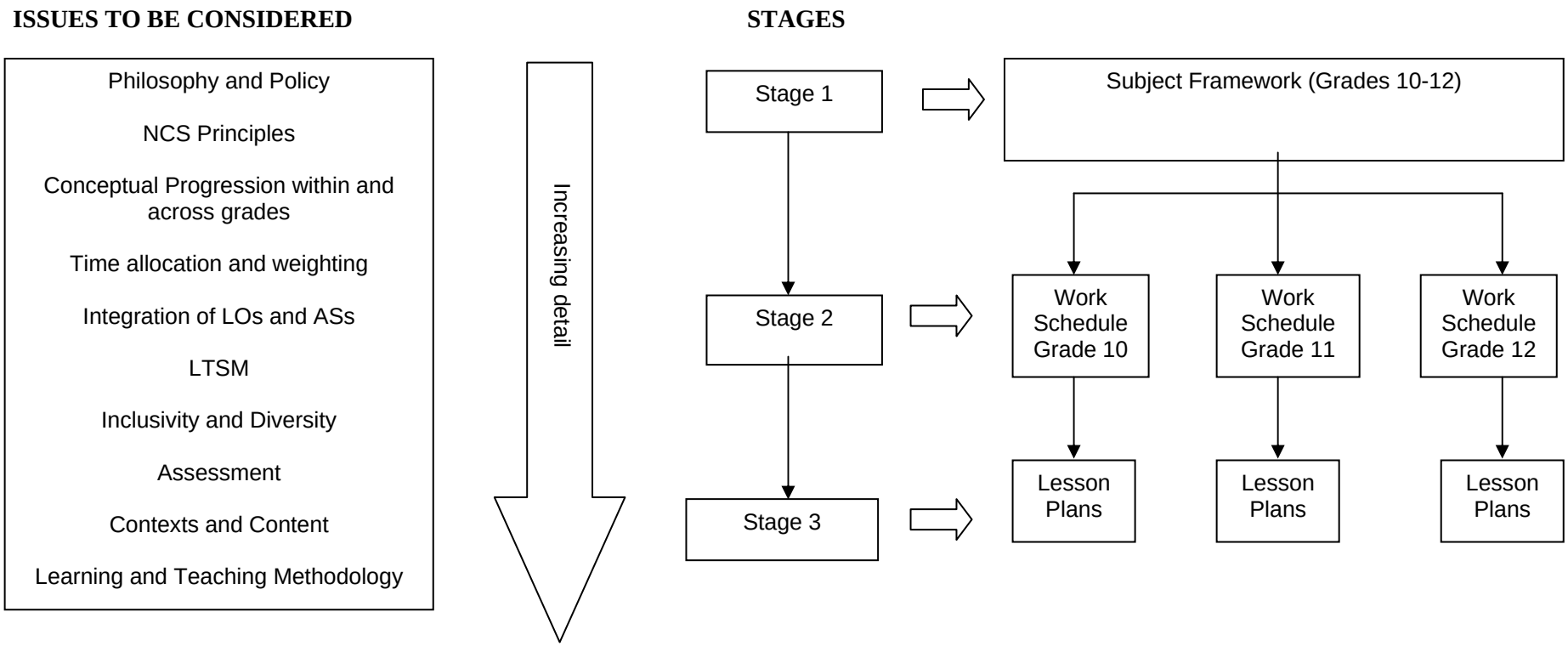
Designing Work Schedules

This is the second phase in the design of a Learning Programme. In this phase teachers develop Work Schedules for each grade. The Work Schedules are informed by the planning undertaken for the Subject Framework. The Work Schedules should be carefully prepared documents that reflect what teaching and assessment will take place in the 36-40 weeks of the school year.

Designing Lesson Plans

Each grade-specific Work Schedule must be divided into units of deliverable learning experiences, that is, Lesson Plans. Lesson Plans are not equivalent to periods in the school timetable. Each Lesson Plan should contain a coherent series of teaching, learning and assessment activities. A Lesson Plan adds to the level of detail for each issue addressed in the Work Schedule. It also indicates other relevant issues to be considered when teaching and assessing a subject.

FIGURE 1: RELATIONSHIP BETWEEN THE 3 STAGES OF PLANNING WHEN DEVELOPING A LEARNING PROGRAMME



SECTION 2

INTRODUCING MATHEMATICS

2.1 WHAT IS MATHEMATICS?

The curriculum for the subject Mathematics is based on the following view of the nature of the discipline.

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of mathematics itself. It is a distinctly human activity practised by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns, which, with rigorous logical thinking, leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives.

The Subject Statement for Mathematics Grades 10-12 expects the learner to expand on the understanding of probability and chance. It requires the learner to collect, organise, analyse and interpret data to establish statistical and probability models and to solve related problems with a focus on human rights issues, inclusivity, current matters involving conflicting views, and environmental and health issues.

2.2 WHAT IS THE PURPOSE OF MATHEMATICS?

2.2.1 Why is Mathematics valuable?

Mathematics provides powerful conceptual tools to:

- Analyse situations and arguments;
- Make and justify critical decisions; and
- Take transformative action, thereby empowering people to:
    - Work towards the reconstruction and development of society.
    - Develop equal opportunities and choice.
    - Contribute towards the widest development of society's cultures, in a rapidly changing technological global context.
    - Derive pleasure and satisfaction through the pursuit of rigour, elegance and the analysis of patterns and relationships.
    - Engage with political, organizational and socio-economic relations.

2.2.2 Educational and career links

Mathematics is an essential element in the curriculum of any learner who intends to pursue a career in the Physical, Mathematical, Computer, Life, Earth, Space and Environmental Sciences or in Technology. Mathematics also has an important role in the Economic, Management, and Social Sciences. It is an important tool for creating, exploring and expressing theoretical and applied aspects of the sciences. Mathematics is also important for the personal development of any learner. Mathematics is used as a tool for solving problems related to modern society and for accelerating development in societies and economies. The financial aspects of dealing with daily life are informed by mathematical considerations. Mathematical ways of thinking are often evident in the workplace.

2.3 WHAT IS THE RELATIONSHIP BETWEEN MATHEMATICS AND THE NATIONAL CURRICULUM STATEMENT PRINCIPLES?

The Constitution of the Republic of South Africa (Act 108 of 1996) provided a basis for curriculum transformation and development in South Africa. The National Curriculum Statement Grades 10-12 (General) lays a foundation for the achievement of these goals by stipulating Learning Outcomes and Assessment Standards, and by spelling out the key principles and values that underpin the curriculum. The Mathematics curriculum supports the application of the nine NCS principles as follows:

2.3.1 Social Transformation

Mathematics and Mathematical Literacy are of fundamental importance to the building of a competitive nation and a thriving democracy. All learners in Grades 10-12 should be given the opportunity of developing themselves mathematically. All learners offering the subject are empowered by the emphasis on the development of process skills as opposed to rote learning of procedures and manipulative skills.

2.3.2 Outcomes-Based Education

The teaching and learning of Mathematics works towards the attainment of the 12 Critical and Developmental Outcomes. These Guidelines will assist the teacher in developing Learning Programmes, which support the achievement of these 12 outcomes.

2.3.3 High knowledge and high skills

The process skills developed in Mathematics are those that enable learners to become mathematicians as opposed to stunting their growth through an emphasis on rote approaches to the subject. Concepts and "big ideas", key to mathematics, are progressively developed and ensure the mathematical future of the learner. The Assessment Standards indicate the minimum requirements to be attained while the system for grading performance allows for the extension to higher levels of knowledge and skills.

2.3.4 Integration and applied competence

An integrated understanding of mathematical concepts is provided for in the holistic view of the Learning Outcomes, as well as in the requirement that learners use existing knowledge and understanding to solve problems as a basis for further development. The emphasis on mathematical modelling in dealing with real life contexts empowers the learner to apply the mathematics learned.

2.3.5 Progression

In developing the Assessment Standards care has been taken to provide for conceptual and skill progression from grade to grade. This is evident in the layout of these standards in the Subject Statement.

2.3.6 Articulation and portability

Any learner offering Mathematics in Grades 10-12 will be working towards the same Learning Outcomes. A learner who achieves the Assessment Standards for Mathematics will be well prepared for the mathematics required by Higher Education Institutions.

2.3.7 Human rights, inclusivity and environmental and social justice

To ensure that human rights issues are emphasized, teachers are encouraged to develop lesson plans that include appropriate contexts. Quantitative arguments are often used in arguing for and against developments that could affect the environment negatively. Appropriate Assessment Standards have been included which challenge learners to critique such arguments. Learning Outcome 4 (Data Handling) is particularly suited to dealing with statistics on a wide range of issues of social justice and in appreciating the threat posed by HIV-AIDS.

2.3.8 Valuing Indigenous Knowledge Systems

Learners in Grades 10-12 come from the many cultures that make up the school-going population of South Africa and must be made aware of the mathematics that is embedded in these cultures. The local environment, for example, local artefacts and architecture, should be studied from a mathematical perspective. Ethnomathematics in South Africa and beyond contributes to the growing body of knowledge in this area.

When learners bring their existing knowledge to bear in problem solving and communicating about mathematics, Indigenous Knowledge Systems will be accessed. In particular, the ability of African languages to describe mathematical concepts should be extended. An example of the descriptive power of indigenous languages is the isiZulu phrase "*pinda-pinda*" for multiplication. There are many more examples.

2.3.9 Credibility, quality and efficiency

The scope provided by the Assessment Standards and the extended opportunities that can be realised by higher gradings in attainment ensure that Mathematics is on a par with the mathematics taught in corresponding grades internationally. The formative aspects of Continuous Assessment are intended to enhance the efficiency of teaching and learning. External summative assessment will have a strong role in ensuring the credibility of Mathematics in Grades 10-12.

2.4 PROFILE OF A MATHEMATICS LEARNER

2.4.1 Links between Mathematics in NCS Grades 10-12 and NCS Grades R-9

The learning achieved in Mathematics in Grades R-9 provides an essential base from which to proceed to the demands of Mathematics in Grades 10-12. The essentials of Mathematics developed in Grades R-9 are taken further into working in more symbolic ways. The engagement with space and shape becomes more formalized in Grades 10-12. The methods and uses of statistics and chance are dealt with in greater depth than was the case in Grades R-9. How Mathematics can contribute to an understanding of financial issues goes beyond dealing with budgets. The emphasis on contexts and integration within Mathematics and across the curriculum is maintained while mathematical modelling becomes more prominent.

Table 2.1 shows that the Learning Outcomes in the NCS Grades R-9 and the NCS Grades 10-12 are similar. In planning the elements of a Mathematics Learning Programme it is important to take into account what was covered in Grades R-9.

Table 2.1: How the Mathematics Learning Outcomes of the NCS Grades R-9 and the NCS Grades 10-12 are linked

LEARNING OUTCOME	NCS Grades R-9	NCS Grades 10-12
1	Number and Number Relationships	Number and Number Relationships
2	Patterns, Functions and Algebra	Functions and Algebra
3	Shape and Space	Shape, Space and Measurement
4	Data Handling and Probability	Data Handling and Probability
5	Measurement	

2.4.2 Expectations of learners exiting Grade 12 with Mathematics

The Learning Programme for Grades 10, 11 and 12 should be developed jointly by the Mathematics teachers teaching these grades. The Learning Programme devised should ensure that all the Learning Outcomes and Assessment Standards can be achieved. This implies that learners will be working towards being able to:

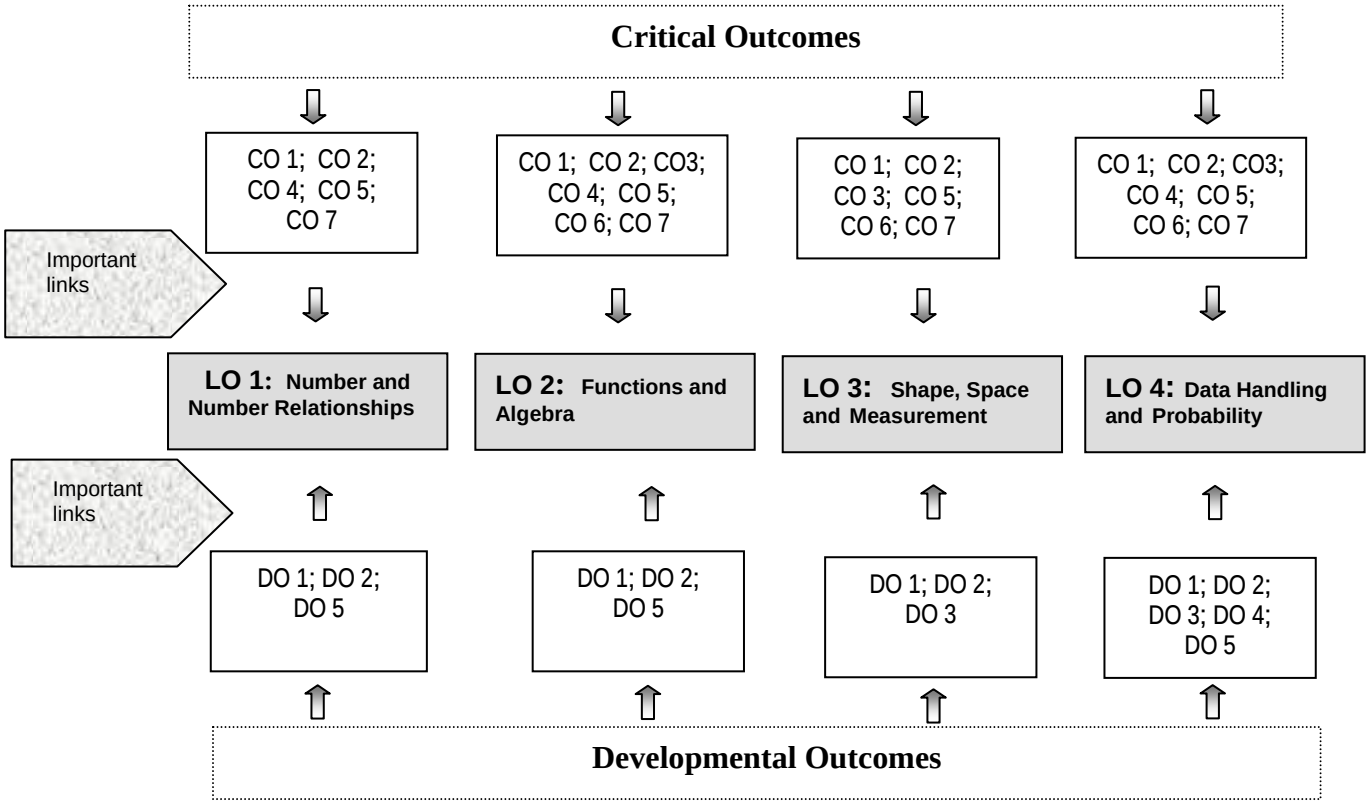
- competently use mathematical process skills such as making conjectures, proving assertions and modelling situations;
- calculate confidently and competently with and without calculators, and use rational and irrational numbers with understanding;
- competently produce useful equivalents for algebraic expressions, and use such equivalents appropriately and with confidence;
- use mathematics to critically investigate and monitor the financial aspects of personal and community life;
- use mathematics to critique arguments on which political decisions are based;
- work with a wide range of patterns and transformations (translations, rotations, reflections) and solve related problems;
- describe, represent and analyse shape and space in two and three-dimensions using geometry and trigonometry;
- collect and use data to establish basic statistical and probability models, solve related problems and critically consider representations provided or conclusions reached;
- use and understand the principles of the differential calculus to determine the rate of change of a range of simple, non-linear functions and solve optimisation problems;
- solve problems involving sequences and series in real life and mathematical situations; and
- use available technology in calculations and in the development of mathematical models.

2.5 RELATIONSHIP BETWEEN MATHEMATICS LEARNING OUTCOMES AND CRITICAL AND DEVELOPMENTAL OUTCOMES

Education and Training in South Africa has 7 Critical Outcomes and 5 Developmental Outcomes, derived from the Constitution. These Critical Outcomes should therefore be reflected in the teaching approaches and methodologies that Mathematics teachers use. Both teachers and learners should be aware of and focus on these Critical and Developmental Outcomes, which will be addressed through Mathematics teaching and learning.

Figure 2.1 illustrates the relationship between the Mathematics Learning Outcomes and the Critical and Developmental Outcomes.

Figure 2.1: Designing down from the Critical Outcomes



2.6 WAYS TO ACHIEVE MATHEMATICS LEARNING OUTCOMES

2.6.1 Approaching the teaching and learning of Mathematics

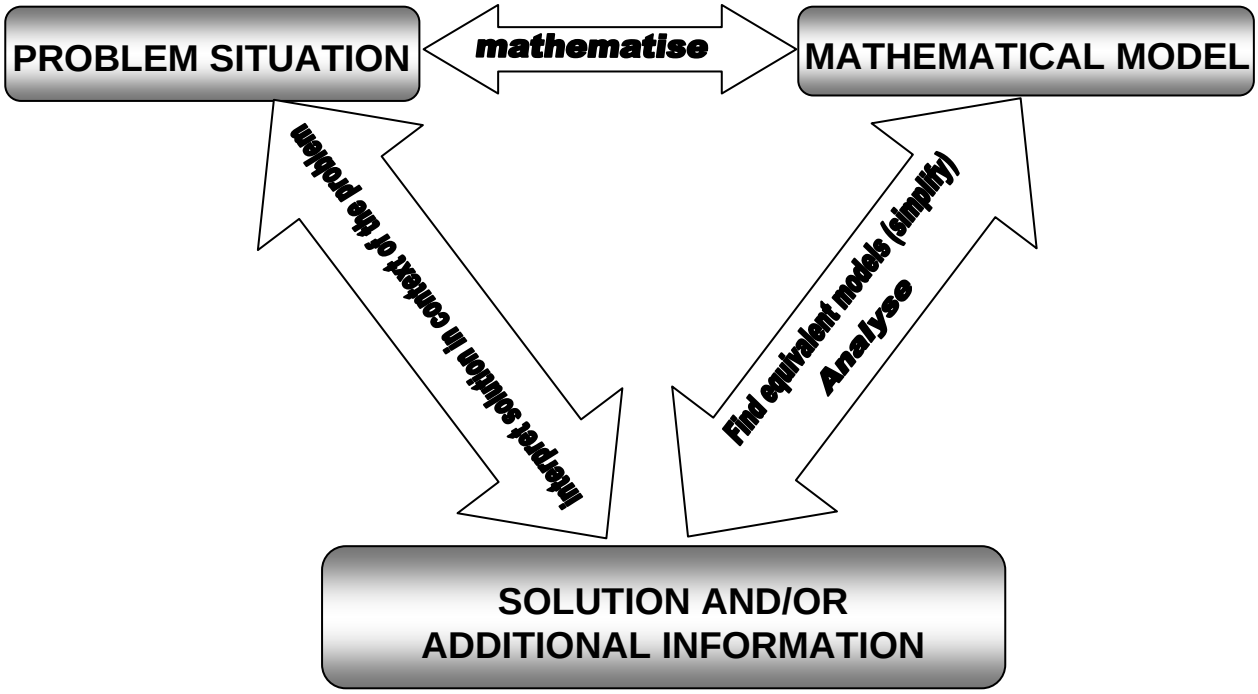
Teaching and learning in mathematics focuses on the development of learners towards the four Learning Outcomes through the attainment of the Assessment Standards for each Grade. Central to the attainment of the Learning Outcomes is the development of mathematical process skills (e.g. investigating, conjecturing, organizing, analysing, proving, problem solving, modelling) Through the use of these skills, knowledge and understanding important concepts in mathematics are built up progressively from Grade to Grade. The teaching and learning is furthermore embedded in contexts that nurture appropriate values and attitudes while relating to real life situations.

Mathematics is a concept and process driven subject. However the teaching and learning of Mathematics cannot be presented in a vacuum. Mathematics should enable learners to establish an authentic connection between mathematics as a discipline and the application of mathematics in real-world contexts.

Mathematical modelling provides learners with a powerful and versatile means of mathematically analysing and describing their world. Mathematical modelling involves identifying and selecting relevant features of real-world situations, representing those features in mathematical representations, quantitatively and qualitatively analysing the model and the characteristics of the situation, and considering the accuracy and limitations of the mathematical model. Mathematical modelling allows learners to deepen their understanding of mathematics while expanding their

repertoire of mathematical tools for solving real-world problems. Figure 2.2 presents a diagrammatic view of the mathematical modelling process.

Figure 2.2 The mathematical modelling process



2.6.2 Interrelatedness of content, processes and contexts

Schools' mathematics instructional programmes need to nurture the development of mathematical proficiency and quantitative literacy in learners. Learning Programmes for Mathematics should arrange selected areas of mathematical content, and the use of mathematical process skills into coherent, comprehensive learning programmes that foster mathematical development over time.

Mathematical content and processes must, where appropriate, be embedded into contexts that will make it possible for learners to establish an authentic connection between mathematics as a discipline and the application of mathematics in real-world contexts.

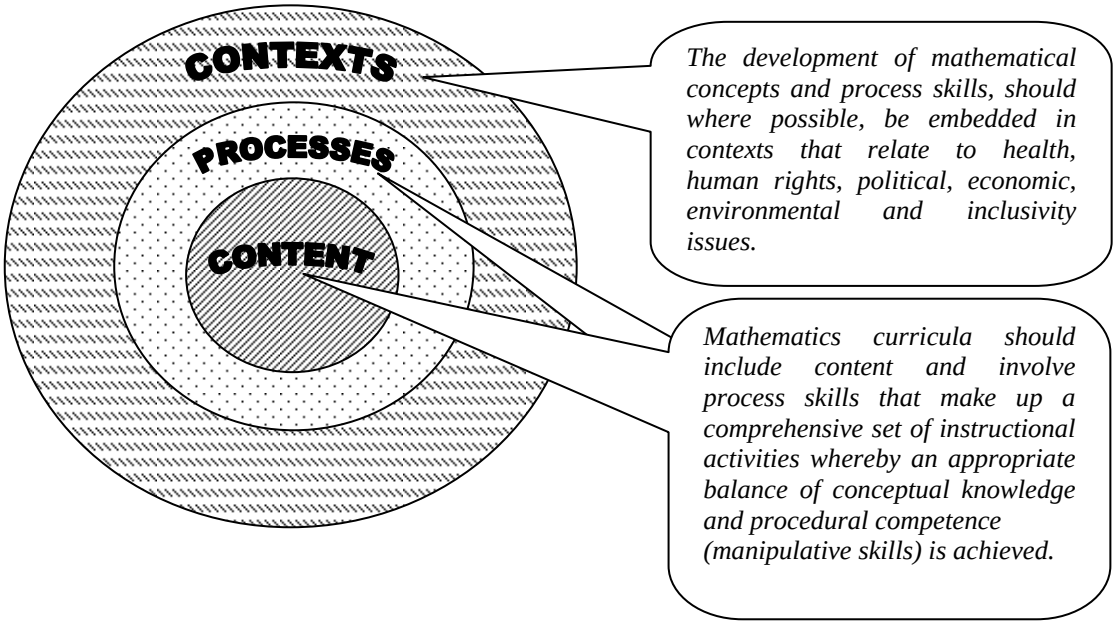
Mathematical content, processes and contexts are important for instructional programmes if:

- they are useful – either for developing other mathematical ideas, linking areas of mathematics, or forming and solving problems within or outside mathematics;
- they assist in the development of appropriate values and attitudes;
- they build productively on learners' prior knowledge and experience, engage their interest, and make sense; and
- they are designed to systematically provide learners with mathematical experiences that become progressively deeper and broader.

The mathematical processes, for example, of collecting data, recognising patterns, formulating conjectures, generalising from these experiences and developing a coherent explanation of one's findings, form the basis of an inquiry approach. For this approach to succeed, it must create a culture of respect for the different kinds of intelligence, varied skills, and diverse backgrounds that learners bring to academic endeavour. It is in this educational environment that students learn to

take intellectual risks and become critical thinkers who will be productive and quantitatively competent citizens. Figure 2.3 illustrates the interrelatedness of content, processes and contexts.

Figure 2.3 Interrelatedness of content, processes and contexts



2.6.3 Mathematics and Technology

Television and the Internet are invaluable sources of information. Multimedia computing offers a world of excitement and interest to the learner. For mathematics, in particular, electronic computational devices are currently at the core of the use of mathematics in everyday life (ATMs for example) and in industry (numerical methods in modelling) and commerce (accounting packages).

The computer together with custom designed software packages has become a powerful aid to developing mathematical concepts. The visualisation made possible through the dynamic capabilities of the computer especially so, not only for teaching and learning, but also for the advancement of mathematics and its applications. Spreadsheets, graphing packages and dynamic geometry software (e.g. Cabri Geometrie, and Geometer's Sketchpad) are particularly useful in this regard.

Many learning sites in South Africa currently have no electricity. The battery powered graphics calculator is a valuable tool which could bring many of the advantages of computational technology to the mathematics classroom. The scientific calculator is indispensable. However, for the successful use of any of the electronic media in education, the teacher needs to be appropriately trained.

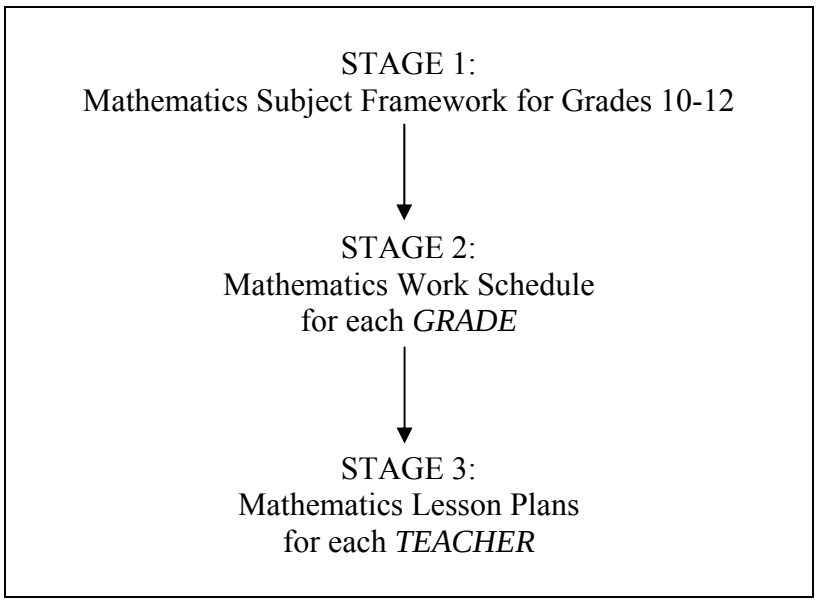
SECTION 3

DESIGNING A LEARNING PROGRAMME FOR MATHEMATICS

3.1 INTRODUCTION

A Learning Programme is a tool to plan for sequenced learning, teaching and assessment across Grades 10-12 so that all four Learning Outcomes in Mathematics are achieved in a progressive manner. It is recommended that the Mathematics teachers at a school first put together a broad subject outline (i.e. Subject Framework) for Grades 10-12 to arrive at an understanding of the progression which needs to take place across the grades (see Section 3.3.1). This will assist with the demarcation of content for each grade. Thereafter, Mathematics teachers teaching the same grade need to work together and draw from the content and context identified for their grade in the Subject Framework, to develop a Work Schedule in which they indicate the sequence in which the content and context will be presented for Mathematics in that particular grade (see Section 3.3.2). Finally, the individual Mathematics teacher should design Lesson Plans using the grade-specific Work Schedule as the starting point. The Lesson Plans should include learning, teaching and assessment activities (see Section 3.3.3).

An outline of the process involved in the design of a Learning Programme for Mathematics is provided in the diagram below:



The process to be followed in the development of a Learning Programme is not a neatly packaged sequence of numbered steps that follow one another in a particular order. Teachers may find themselves moving back and forth in the process as they plan and critically reflect on decisions taken before moving on to the next decision in the process. The process is therefore not strictly linear and is reflective in nature. For this reason the steps provided in this Section are a guide and should be used as a checklist in the planning process.

3.2 ISSUES TO ADDRESS WHEN DESIGNING A LEARNING PROGRAMME

The issues to be addressed in the development of a Mathematics Learning Programme are presented in a tabular format to indicate the implications of each issue at each of the three stages of the development of a Learning Programme:

- Stage 1 – Subject Framework
- Stage 2 – Work Schedule
- Stage 3 – Lesson Plan

3.2.1 Policies and Principles

STAGE 1 **Subject Framework** **STAGE 2** **Work Schedule** **STAGE 3** **Lesson Plan**	The various Policies that impact on curriculum implementation should be considered throughout the planning process. *NCS:* • Principles: Refer to Section 2.3 to see how Mathematics supports the application of the nine principles of the NCS • Critical and Developmental Outcomes: Refer to Section 2.5 to see how Mathematics supports the application of the Critical and Developmental Outcomes *Other Policies and Legislation:* • White Paper 6, Language in Education Policy, Religion and Education Policy, HIV/AIDS Policy– all have implications for LTSM and teaching methods in Mathematics • White Paper 7 – gives an indication on the use of computers in the classroom and therefore has implications for LTSM and teaching methods in Mathematics

3.2.2 Content

In the NCS Grades 10-12 content means the combination of knowledge, skills and values.

STAGE 1 **Subject Framework**	The content is provided by the ASs. These give an indication of the knowledge, skills and values (KSVs) to be covered in each of the three grades. The Subject Framework sets out the content for the three years (i.e. Grades 10, 11 and 12).
STAGE 2 **Work Schedule**	The Work Schedule sets out the content for one year. Here the focus falls on the grade-specific KSVs required by the NCS.
STAGE 3 **Lesson Plan**	The Lesson Plans set out the content to be covered in each coherent series of learning, teaching and assessment activities. Each Lesson Plan can be one or more weeks in duration.

3.2.3 Integration

Integration involves the grouping of Assessment Standards according to natural and authentic links.

STAGE 1 **Subject Framework**	Integration within the subject should be considered in broad terms during discussions at this stage. All Grade 10-12 teachers should consider integration of ASs within and across the grades.
STAGE 2 **Work Schedule**	The integration and sequencing of the ASs is undertaken in the Work Schedule to ensure that all ASs for a particular grade are covered in the 40-week contact period.
STAGE 3 **Lesson Plan**	The same groupings of LOs and ASs as arrived at in the Work Schedule should be used to develop a coherent series of learning, teaching and assessment activities for each Lesson Plan.

3.2.4 Conceptual Progression

STAGE 1 **Subject Framework**	The Subject Framework should indicate the increasing depth of difficulty across Grades 10-12. Progression across the three grades is shown in the ASs per Learning Outcome.
STAGE 2 **Work Schedule**	Progression in a grade is evident in the increasing depth of difficulty in that particular grade. Grade-specific progression is achieved by appropriately sequencing the groupings of integrated LOs and AS in the Work Schedule.
STAGE 3 **Lesson Plan**	In the individual Mathematics classroom increasing depth of difficulty is shown in the activities and Lesson Plans. Progression is achieved by appropriately sequencing the activities contained within each Lesson Plan and in the series of Lesson Plans.

3.2.5 Time Allocation and Weighting

STAGE 1 **Subject Framework**	4 hours per week is allocated to Mathematics in the NCS. This is approximately 160 hours per year. The teachers of the subject should plan how this time will be used for the teaching of Mathematics in the three grades.
STAGE 2 **Work Schedule**	The groupings of ASs as arrived at in the integration process should be paced across the 40 weeks of the school year to ensure coverage of the curriculum.
STAGE 3 **Lesson Plan**	The amount of time to be spent on activities should be indicated in the Lesson Plans.

3.2.6 LTSM

LTSM refers to any materials that facilitate learning and teaching. LTSM need to be chosen judiciously because they have cost implications for the school and the learner. The NCS provides scope for the use of a variety of resources. All teachers and learners must have a textbook. However, teachers are required to go beyond the textbook. They do not necessarily need exotic, specialised materials. Rather common and readily available items can be used.

STAGE 1 **Subject Framework**	Compile a list of general LTSM (text books and other resources) that will be necessary and useful in the teaching, learning and assessment of the content. This assists with the requisition and availability of LTSM at a school.
STAGE 2 **Work Schedule**	List grade-specific LTSM (resources) required in the learning, teaching and assessment process for the grade.
STAGE 3 **Lesson Plan**	Identify specific resources related to the individual activities contained within a Lesson Plan.

3.2.7 Assessment

All Grade 10 and 11 learners are expected to complete eight internal tasks for Mathematics. Of the eight tasks, two must be tests, two must be examinations and the remaining four tasks can take any form suitable to the teaching and assessment of Mathematics. Grade 12 learners are expected to complete seven internal tasks, including two tests and two examinations. The remaining three tasks can take any form suitable to the teaching and assessment of Mathematics. In addition, Grade 12 learners are expected to complete an external examination. See Section 3 of the Subject Assessment Guidelines for Mathematics for further information.

In order to administer effective assessment one must have a clearly defined purpose. It is important that all the tasks are well covered as spelt out in the Subject Assessment Guideline document. By answering the following questions the teacher can decide what assessment activity is most appropriate:

- What concept, skill or knowledge needs to be assessed?
- What should the learners know?
- At what level should the learners be performing?
- What type of knowledge is being assessed: reasoning, memory or process?

Observation-based assessment requires that learner performance be assessed while the learner is actually performing a skill in the classroom as there will be no concrete product for the teacher to assess after the performance. Not all observations need culminate in a formally recorded assessment of learner performance. **Performance-based** assessment relies on the availability of a product as evidence of learner performance that can be assessed by the teacher after the completion of the performance. **Test-based** assessment focuses on assessing the presentation and application of knowledge.

STAGE 1 Subject Framework	Develop a three-year assessment plan using the Subject Assessment Guidelines for Mathematics. This should ensure the use of a variety of assessment forms relevant to the subject and progression across the three grades.
STAGE 2 Work Schedule	Use the Subject Assessment Guidelines for Mathematics to develop a grade-specific assessment plan. The forms of assessment listed must facilitate the achievement of the particular LOs and ASs in each grouping.
STAGE 3 Lesson Plan	Indicate more classroom-specific assessment strategies, by mentioning the methods, forms and tools that will be used to assess learner performance in each activity. HINT: Not all activities need to be assessed – some may just be introductory in nature or for enrichment. The choice of an assessment strategy is determined by the LOs and ASs that have been grouped together for a particular Lesson Plan. The assessment strategy chosen must facilitate the achievement of these particular LOs and ASs in the classroom.

3.2.8 Inclusivity and Diversity

The following steps can be taken to effectively address diversity in the classroom when planning Mathematics teaching activities:

- consider individual past experiences, learning styles and preferences;
- develop questions and activities that are aimed at different levels of ability;
- provide opportunity for a variety of participation levels such as individual, pairs and small group activities;
- consider the value of individual methods ; and
- assess learners based on individual progress.

STAGE 1 Subject Framework **STAGE 2 Work Schedule**	Teachers should be sensitive to inclusivity and diversity when identifying content, teaching styles and methods, forms of assessment and LTSM (Resources). Diversity should be accommodated in the following areas: • Learning styles: provide optional activities / different ways of doing same activity • Pace of learning: provide for both slower and faster learners by providing optional extra activities, reading or research, as well as multiple assessment opportunities • Differences in levels of achievement: provide optional extra activities, challenges and materials that cater for these differences between learners. • Gender diversity: ensure that teachers do not inadvertently allow or contribute towards discrimination against boys or girls in the classroom on the basis of gender. • Cultural diversity: recognise, celebrate and be sensitive when choosing content, assessment tasks and LTSM.
STAGE 3 Lesson Plan	This is catered for as EXPANDED OPPORTUNITIES in the Lesson Plan. Enrichment is provided for high achievers and remediation or other relevant opportunities for learners requiring additional support. It is not necessary to develop an activity to cater for each type of diversity which arises in the classroom. Teachers may find it possible to cater for different diversities within one activity with effective planning.

3.2.9 Learning and Teaching Methodology

STAGE 1 Subject Framework **STAGE 2 Work Schedule**	It is not necessary to record Teaching Methods for either of these stages.
STAGE 3 Lesson Plan	This is catered for as TEACHING METHOD in the Lesson Plan. It provides an indication of how teaching and learning will take place, that is, how each activity will be presented in the classroom.

3.3 DESIGNING A LEARNING PROGRAMME

A detailed description of the process involved in the design of a Learning Programme for Mathematics is provided in this section (see Sections 3.3.1 – 3.3.3). The process presented here is a suggestion of how to go about designing a Learning Programme.

3.3.1 Subject Framework (Grades 10-12) for Mathematics

Planning for the teaching of Mathematics in Grades 10 to 12 should begin with a detailed examination of the scope of the subject as set out in the Subject Statement. No particular format or template is recommended for this first phase of planning but the five steps below should be used as a checklist.

Although no prescribed document is required for this stage of planning, school-wide planning (timetables, ordering, teacher development, classroom allocation) as well as the development of grade-specific work schedules would benefit from short documents which spell out:

- The scope of the subject – the knowledge, skills and values; the content; the contexts or themes; electives etc. to be covered in the three grades
- A three-year assessment plan
- The list of LTSM required

❶ Clarify the Learning Outcomes and Assessment Standards.

The essential question for Mathematics is: What Learning Outcomes do learners have to master by the end of Grade 12 and what Assessment Standards should they achieve to show that they are on their way to mastering these outcomes?

All learning, teaching and assessment opportunities must be designed down from what learners should know, do and produce by the end of Grade 12. The Learning Outcomes and Assessment Standards that learners should master by the end of Grade 12 are specified in the Mathematics Subject Statement.

❷ Study the conceptual progression across the three grades.

Study the Assessment Standards for Mathematics across the three grades. Progression should be clearly evident across the grades.

❸ Identify the content to be taught.

Analyse the Assessment Standards to identify the skills, knowledge and values to be addressed in each grade. Also consider the content and context in which they will be taught.

❹ Identify three-year plan of assessment.

Use the Subject Assessment Guidelines to guide the three-year assessment plan. Consider what forms of assessment will be best suited to each of the Learning Outcomes and Assessment Standards. This ensures that assessment remains an integral part of the learning and teaching process in Mathematics and that learners participate in a range of assessment activities.

❺ Identify possible LTSM (resources).

Consider which LTSM will be best suited to the learning, teaching and assessment of each Learning Outcome in the three grades using the Assessment Standards as guidance.

3.3.2 Designing Work Schedules for Mathematics

This is the second phase in the design of a Learning Programme. In this phase teachers develop Work Schedules for each grade. The Work Schedules are informed by the planning undertaken for the Subject Framework. The Work Schedules should be carefully prepared documents that reflect what teaching and assessment will take place in the 40 weeks of the school year. See Annexure 1 for extracts of Work Schedules for Grades 10, 11 and 12 and Annexures 2, 3 and 4 for examples of Work Schedules for Grades 10, 11 and 12.

The following steps provide guidelines on how to approach the design of a Work Schedule per grade for Mathematics:

❶ Package the content.

Study the Learning Outcomes and Assessment Standards prescribed for the particular grade in Mathematics and group these according to natural and authentic links.

❷ Sequence the content.

Determine the order in which the groupings of Learning Outcomes and Assessment Standards will be presented in the particular grade in Mathematics. Besides the conceptual progression in the Assessment Standards for Mathematics, *context* can also be used to sequence groupings in Mathematics.

❸ Pace the content.

Determine how much time in the school year will be spent on each grouping of Learning Outcomes and Assessment Standards in the particular grade.

❹ Review forms of assessment.

Revisit the forms of assessment listed for the particular grade in the Subject Assessment Guidelines, and refine them to address each grouping of Learning Outcomes and Assessment Standards as developed in Step 1.

❺ Review LTSM.

Revisit the LTSM (resources) listed for the particular grade in the Subject Framework, and refine them to address each grouping of Learning Outcomes and Assessment Standards as developed in Step 1.

3.3.3 Designing Lesson Plans for Mathematics

Each grade-specific Work Schedule for MATHEMATICS must be divided into units of deliverable learning experiences, that is, Lesson Plans. A Lesson Plan adds to the level of detail in the Work Schedule. It also indicates other relevant issues to be considered when teaching and assessing Mathematics.

A Lesson Plan is not equivalent to a subject period in the school timetable. Its duration is dictated by how long it takes to complete the coherent series of activities contained in it.

❶ Indicate the content, context, Learning Outcomes and Assessment Standards.

Copy this information from the Work Schedule for the particular grade.

❷ Develop activities and select teaching method.

Decide how to teach the Learning Outcomes and Assessment Standards indicated in Step 1 and develop the activity or activities that will facilitate the development of the skills, knowledge and values in the particular grouping. Thereafter, determine the most suitable teaching method(s) for the activities and provide a description of how the learners will engage in each activity.

❸ **Consider diversity.**

Explore the various options available within each activity that will allow expanded opportunities to those learners that require individual support. The support provided must ultimately guide learners to develop the skills, knowledge and values indicated in the grouping of Learning Outcomes and Assessment Standards.

❹ **Review assessment and LTSM.**

Indicate the details of the assessment strategy and LTSM to be used in each activity.

❺ **Allocate time.**

Give an indication of how much time will be spent on each activity in the Lesson Plan.

3.3.4 Reflection and review of the Mathematics Learning Programme

After the Learning Programme has been delivered by means of Lesson Plans in the classroom, the teacher should reflect on what worked, how well it worked and what could be improved. Teachers need to note these while the experience is still fresh in their minds, so that if necessary, they can adapt and change the affected part of the Mathematics Learning Programme for future implementation. It is advisable to record this reflection on the Lesson Plan planning sheets.

ANNEXURE 1: EXTRACTS FROM WORK SCHEDULES FOR MATHEMATICS

An extract from a possible Work Schedule for Mathematics Grade 10 (first five weeks only)

<table>
<tr><th>Weeks</th><th>Topic</th><th>Lesson focus and activities</th><th>Learning Outcome(s)</th><th>Assessment Standard(s)</th><th>Assessment Task</th></tr>
<tr><td>1</td><td rowspan="4">Functions and their graphs</td><td>Conversion between numerical, graphical, verbal and symbolic representations. (p. 48 Subject Statement)
• Develop formulas for different tables of values (supplied)
• Develop formulas for relationships described in words</td><td rowspan="4">LO2: Functions and Algebra
(The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems)</td><td rowspan="4">10.2.1 (a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard.
AS: Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).

10.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters a and q on the graphs of functions including:
$y = ax + q$
$y = ax^2 + q$
$y = \frac{a}{x} + q$
$y = ab^x + q;\ b > 0$
$y = a\sin(x) + q$
$y = a\cos(x) + q$
$y = a\tan(x) + q$

10.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed in 10.2.2 above:
(a) domain and range;
(b) intercepts with the axes;
(c) turning points, minima and maxima;
(d) asymptotes;
(e) shape and symmetry;
(f) periodicity and amplitude;
(g) average gradient (average rate of change);
(h) intervals on which the function increases/decreases;
(i) the discrete or continuous nature of the graph.</td><td rowspan="2">Assignment</td></tr>
<tr><td>2</td><td>Conversion between numerical, graphical, verbal and symbolic representations. (p. 48 Subject Statement)
• Develop tables of values for each of the functions starting with a = 1 and q = 0
• Plot graphs for each of the functions from the tables of values</td></tr>
<tr><td>3 and 4</td><td>Investigation of the effects of the parameters a and q on the above functions. (p. 48 Subject Statement)
• Develop tables of values and plotting the graphs for each of the functions changing the value of a and keeping $q = 0$
• Develop tables of values and plotting the graphs for each of the functions changing the value of q and keeping $a = 1$</td><td rowspan="2">Investigation</td></tr>
<tr><td>5</td><td>Describe the graphs of the functions in terms of the characteristics listed in Assessment Standard 10.2.2</td><td>Test</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

An extract from a possible Work Schedule for Mathematics Grade 11

<table>
<tr><th>Weeks</th><th>Topic</th><th>Lesson focus and activities</th><th>Learning Outcome(s)</th><th>Assessment Standard(s)</th><th>Assessment Task</th></tr>
<tr><td>8, 9</td><td rowspan="2">Co-ordinate Geometry</td><td>Revision of Grade 10 work:
• General equation and properties (gradient) of a straight line (y = a x + q)
• Plotting points on a Cartesian co-ordinate system
• Use of the formula for calculating the gradient of a line segment joining two points (see Grade 10 LO3, AS 10.3.3)

Use grid paper to investigate the relationship between the gradients of:
• parallel lines
o provide graph paper with a large number of lines drawn on it
o calculate the gradient of each line
o identify pairs of lines that are parallel
o investigate a relationship between the gradients of the parallel lines
• perpendicular lines
o use same step as parallel lines.

Use grid paper to investigate the inclination of lines i.e. show/derive the formula:
gradient = tan (θ)
where θ is the angle that the line makes with the positive x-axis

Derive and apply the equation the lines listed in Assessment Standard 11.3.3 using:
• $y - y_1 = m(x - x_1)$, and generalise to:
• $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \times (x - x_1)$; where $x_2 \neq x_1$</td><td rowspan="2">Learning Outcome 3: Space, Shape and Measurement
The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.</td><td>11.3.3 Use a Cartesian co-ordinate system to derive and apply:
(a) the equation of a line through two given points
(b) the equation of a line through one point and parallel or perpendicular to a given line
(c) the inclination of a line.</td><td rowspan="2">Investigation</td></tr>
<tr><td>10</td><td>Use grid paper to investigate the relationship between a point A (x_A; y_A) and its image B (x_B; y_B) after:
• rotation around the origin through and angle of 90º or 180º
• an enlargement through the origin by a scale factor k
• collect data for a large number of different points A and B

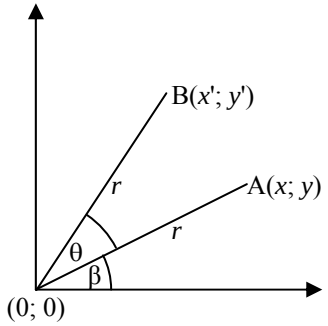
Generalise and observe that the image of A (x_A; y_A) is the following:
• B (x_B; y_B) = ($-y_A$; x_A) for a rotation of 90º
• B (x_B; y_B) = ($-x_A$; $-y_A$) for a rotation of 180º
• B (x_B; y_B) = (kx_A; $-ky_A$) for an enlargement through the origin, by a constant factor k.</td><td>11.3.4 Investigate, generalise and apply the effect on the co-ordinates of:
(a) the point (x ; y) after rotation around the origin through an angle of 90º or 180º;
(b) the vertices (x1; y1), (x2; y2)…, (xn; yn) of a polygon after enlargement through the origin, by a constant factor k.</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

An extract from a possible Work Schedule for Mathematics Grade 12

<table>
<tr><th>Weeks</th><th>Topic</th><th>Lesson focus and activities</th><th>Learning Outcome(s)</th><th>Assessment Standard(s)</th><th>Assessment Task</th></tr>
<tr><td>15</td><td>Co-ordinate Geometry of circles</td><td>Introduce, practice using and apply:
• the equation for the equation of a circle (any centre)
• the equation of a tangent to a circle given a point on the circle</td><td rowspan="2">Learning Outcome 3: Space, Shape and Measurement
The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.</td><td>12.3.3 Use a two-dimensional Cartesian co-ordinate system to derive and apply:
(a) the equation of a circle (any centre);
(b) the equation of a tangent to a circle given a point on the circle.

NOTE learners are expected to know and be able to use as an axiom: “the tangent to a circle is perpendicular to the radius drawn to the point of contact”</td><td rowspan="2">Test</td></tr>
<tr><td>16 and 17</td><td>The trigonometry of rotations</td><td>Recall the following from Grade 11:
The image of A (x_A; y_A) is:
• B (x_B; y_B) = ($-y_A$; x_A) for a rotation of 90º
• B (x_B; y_B) = ($-x_A$; $-y_A$) for a rotation of 180º
• B (x_B; y_B) = (kx_A; $-ky_A$) for an enlargement through the origin, by a constant factor k.

Introduce the compound angle identities (without derivation)

Develop a general formula for the image of A (x_A; y_A) after a rotation about the origin through an angle θ (see derivation below)
 

By basic trigonometry definitions:
$x = r \times \cos\beta$ and $y = r \times \sin\beta$ (1)
And $x' = r \times \cos(\theta + \beta)$ and
$y' = r \times \sin(\theta + \beta)$
Using the compound angle formulae:
$x' = r \times (\cos\theta \cos\beta - \sin\theta \sin\beta)$
$y' = r \times (\sin\theta \cos\beta + \cos\theta \sin\beta)$
Substituting (1) we get
$x' = x \cos\theta - y \sin\theta$
$y' = x \sin\theta + y \cos\theta$

Revisit the special cases developed in Grade 11 to confirm that the formulae are correct, namely: rotation about the origin through 90˚ and 180˚</td><td>12.3.4
(a) Use the compound angle identities to generalise the effect on the co-ordinates of the point (x; y) after rotation about the origin through an angle.
(b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that enlargement preserves shape but not size.

12.3.5 Use the following compound angle identities (without derivation):
(a) $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$
(b) $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$
(c) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

ANNEXURE 2: EXAMPLE OF A GRADE 10 WORK SCHEDULE FOR MATHEMATICS

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
1	**Rational numbers** Conversions between terminating or recurring decimals and the form: $\frac{a}{b}; a,b \in Z; b \neq 0$	10.1.1 Identify rational numbers and convert between terminating or recurring decimals and the form: $\frac{a}{b}; a,b \in Z; b \neq 0$	Note: A textbook for each learner is essential. Scientific calculators.	
2	**Algebraic expressions**	10.2.4 Manipulate algebraic expressions: (a) multiplying a binomial by a trinomial (b) factorising trinomials (c) factorising by grouping in pairs (d) simplifying algebraic fractions with monomial denominators		
3 & 4	**Functions defined by:** • $y = ax + q$ • $y = ax^2 + q$ • $y = \frac{a}{x} + q$	10.2.1 (a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard. (b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).		
5 & 6 & 7	**Graphs**	10.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters *a* and *q* on the graphs of functions including: $y = ax + q$ $y = ax^2 + q$ $y = \frac{a}{x} + q$		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
		10.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed in 10.2.2 above: (a) domain and range (a) intercepts with the axes (b) turning points, minima and maxima (c) asymptotes (d) shape and symmetry (e) periodicity and amplitude (f) average gradient (average rate of change) (g) intervals on which the function increases/decreases (i) the discrete or continuous nature of the graph.		
8 & 9	**Right Prisms** • Surface Area • Volume	10.3.1 Understand and determine the effect on the volume and surface area of right prisms and cylinders, of multiplying any dimension by a constant factor k.		
10 & 11	**Triangles, quadrilaterals and other polygons**	10.3.2 (a) Through investigations, produce conjectures and generalisations related to triangles, quadrilaterals and other polygons, and attempt to validate, justify, explain or prove them, using any logical method (Euclidean, co-ordinate and/or transformation).		
12 & 13	**Exponents and surd** • The laws of exponents for integral exponents. • Rational approximation of surds • Placing a surd between two integers • Round rational and irrational numbers to an appropriate degree of accuracy.	10.1.2 (a) Simplify expressions using the laws of exponents for integral exponents. (b) Establish between which two integers any simple surd lies. (c) Round rational and irrational numbers to an appropriate degree of accuracy.		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
14 & 15	**Triangles, quadrilaterals and other polygons**	**NB: OPTIONAL ASSESSMENT STANDARD 10.3.2** **(a) Disprove false conjectures by producing counter-examples.** **(b) Investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangle, the kite, parallelogram, rectangle, rhombus and square).**		
16 & 17 & 18	**Equations** • Linear • Linear in two variables	10.2.5 Solve: (a) linear equations (b) quadratic equations by factorisation (c) exponential equations of the form $ka^{x+p} = m$ (including examples solved by trial and error) (d) linear inequalities in one variable and illustrate the solution graphically (e) linear equations in two variables simultaneously (numerically, algebraically and graphically)		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
19 & 20	**Data handling** **History of the development of geometry and trigonometry in various cultures**	10.4.1 (a) Collect, organise and interpret univariate numerical data in order to determine: • measures of central tendency (mean, median, mode) of grouped and ungrouped data, and know which is the most appropriate under given conditions; • measures of dispersion: range, percentiles, quartiles, inter-quartile and semi-inter-quartile range. (b) Represent data effectively, choosing appropriately from: • bar and compound bar graphs; • histograms (grouped data); • frequency polygons; • pie charts; • line and broken line graphs. 10.3.7 Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through a project.		
21 & 22	**REVISION AND EXAMINATION**			
23	**Number patterns**	10.1.3 Investigate number patterns (including but not limited to those where there is a constant difference between consecutive terms in a number pattern, and the general term is therefore linear) and hence: (a) make conjectures and generalisations (b) provide explanations and justifications and attempt to prove conjectures. 10.1.6 Solve non-routine, unseen problems		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
24	**Simple and compound growth formulae** ***A=P(1+ni) and A=P(1+i)^n*** ; solving for any variable except in the compound growth formula.	10.1.4 Use simple and compound growth formulae ($A = P(1+ni)$ and $A = P(1+i)^n$ to solve problems, including interest, hire-purchase, inflation, population growth and other real-life problems 10.1.6 Solve non-routine, unseen problems.		
25	**Foreign exchange rates**	10.1.5 Demonstrate an understanding of the implications of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel). 10.1.6 Solve non-routine, unseen problems		
26	**Cartesian co-ordinate system**	10.3.3 Represent geometric figures on a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$ and a formula for calculating: (a) the distance between the two points (b) the gradient of the line segment joining the points (c) the co-ordinates of the mid-point of the line segment joining the points.		
27 & 28	**Issues related to health, social, economic, cultural, political and environmental matters**	10.2.6 Use mathematical models to investigate problems that arise in real-life contexts: (a) making conjectures, demonstrating and explaining their validity; (b) expressing and justifying mathematical generalisations of situations; (c) using the various representations to interpolate and extrapolate; (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters).		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
29	**Functions and graphs**	10.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters *a* and *q* on the graphs of functions including: $y = ab^x + q; b > 0$ $y = a\sin(x) + q$ $y = a\cos(x) + q$ $y = a\tan(x) + q$		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
30 & 31 & 32	**Probability**	**NB: OPTIONAL ASSESSMENT STANDARD FOR PAPER 3.** **10.4.2** **(a) Use probability models for comparing the relative frequency of an outcome with the probability of an outcome.** **(b) Use Venn diagrams as an aid to solving probability problems, appreciating and correctly identifying:** • **the sample space of a random experiment;** • **an event of the random experiment as a subset of the sample space;** • **the union and intersection of two or more subsets of the sample space;** • **P(S) = 1 (where S is the sample space);** • **P(A or B) = P(A) + P(B) - P(A and B) (where A and B are events within a sample space);** • **disjoint (mutually exclusive) events, and is therefore able to calculate the probability of either of the events occurring by applying the addition rule for disjoint events: P(A or B) = P(A) + P(B);** • **complementary events, and is therefore able to calculate the probability of an event not occurring: P(not A) = 1- P(A).**		
33	**Average rate of change of a function between two values of the independent variable**	10.2.7 Investigate the average rate of change of a function between two values of the independent variable, demonstrating an intuitive understanding of average rate of change over different intervals (e.g. investigate water consumption by calculating the average rate of change over different time intervals and compare results with the graph of the relationship).		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
34	**Transformations**	10.3.4 Investigate, generalise and apply the effect of the following transformations of the point (x; y): • a translation of p units horizontally and q units vertically • a reflection in the x-axis, the y-axis or the line y = x.		
35	**Trigonometric functions**	10.3.5 Understand that the similarity of triangles is fundamental to the trigonometric functions $\sin\theta$, $\cos\theta$, and $\tan\theta$, and is able to define and use the functions.		
		10.3.6 Solve problems in two dimensions by using the trigonometric functions ($\sin\theta$, $\cos\theta$, and $\tan\theta$) in right-angled triangles and by constructing and interpreting geometric and trigonometric models (examples to include scale drawings, maps and building plans).		
36 & 37	**Data handling**	**NB: OPTIONAL ASSESSMENT STANDARD FOR PAPER 3.** **10.4.3** **(a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).** **(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.**		
38	**REVISION**			
39 & 40	**EXAMINATION**			

ANNEXURE 3: EXAMPLE OF A GRADE 11 WORK SCHEDULE FOR MATHEMATICS

Grade 11 Mathematics Work Schedule

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
1	**Mathematical Modelling with Quadratic Equations** (including: manipulating algebraic expressions; solving quadratic equations and equations in two unknowns; non-real numbers)	11.2.4 Manipulate algebraic expressions: (a) by completing the square; (b) simplifying algebraic fractions with binomial denominators. 11.2.5 Solve: (a) quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically; (b) equations in two unknowns, one of which is linear and one of which is quadratic, algebraically or graphically. 11.2.6 Use mathematical models to investigate problems that arise in real-life contexts: (a) making conjectures, demonstrating and explaining their validity; (b) expressing and justifying mathematical generalisations of situations; (c) using various representations to interpolate and extrapolate; (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters.) 11.1.1 Understand that not all numbers are real. (This requires the recognition but not the study of non-real numbers.) 11.1.6 Solve non-routine, unseen problems.		
2				
3				
4				
5	**Exponents and Surds** (consolidation of laws etc. as needed in other topics)	11.1.2 (a) Simplify expressions using the laws of exponents for rational exponents. (b) Add, subtract, multiply and divide simple surds (e.g. see Subject Statement) (c) Demonstrate an understanding of error margins.		
6	**Number Patterns** (including simple and compound decay)	11.1.3 Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence: (a) make conjectures and generalisations		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
7		(b) provide explanations and justifications and attempt to prove conjectures. 13.1.4 Use simple and compound decay formulae to solve problems (including straight line depreciation and depreciation on a reducing balance) (link to Learning Outcome 2).		
8		11.1.5 Demonstrate an understanding of different periods of compounding growth and decay (including effective compounding growth and decay and including effective and nominal interest rates). 11.1.6 Solve non-routine, unseen problems.		
9	**Co-ordinate and Transformation Geometry** (making sure to develop links to the transformation of functions in Grade 10 (10.2.2) and Grade 11 (11.1.1))	11.3.3 Use a Cartesian co-ordinate system to derive and apply: (a) the equation of a line through two given points (b) the equation of a line through one point and parallel or perpendicular to a given line (c) the inclination of a line. 11.3.4 Investigate, generalise and apply the effect on the co-ordinates of: (a) the point (x ; y) after rotation around the origin through an angle of 90º or 180º; (b) the vertices of a polygon after enlargement through the origin, by a constant factor k.		
10				
11	**Linear Programming (Mathematical Modelling)** (including the opportunity to consolidate understanding of variables, the meaning of letter symbols, the construction of formulae, awareness of functions of two variables)	11.2.8 (a) Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region. (b) Solve a system of linear equations to find the co-ordinates of the vertices of the feasible region. 11.2.6 Use mathematical models to investigate problems that arise in real-life contexts: (a) making conjectures, demonstrating and explaining their validity; (b) expressing and justifying mathematical generalisations of situations; (c) using various representations to interpolate and extrapolate; (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters.) 11.1.6 Solve non-routine, unseen problems.		
12				
13				
14	**Graphs of Trigonometric Functions** (extension of work done in Grade 10 (10.2.2) and laying the foundation for solving equations (11.3.5 & 11.3.6))	11.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for functions including: (see trigonometric graphs in		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
15		11.2.1 (a) Demonstrate the ability to work with various types of functions including those listed in the following Assessment Standard. Refer to the Subject Statement. (b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).		
16	**Working with Data** (including: measures of central tendency and dispersion, fitting curves to bivariate numerical data – modelling)	11.4.1 (a) Calculate and represent measures of central tendency and dispersion in univariate numerical data by: • five number summary (maximum, minimum and quartiles); • box and whisker diagrams; • ogives; • calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data), and representing results graphically using histograms and frequency polygons. (b) Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health, social, economic, cultural, political and environmental issues).		
17 & 18		11.2.6 Use mathematical models to investigate problems that arise in real-life contexts: (a) making conjectures, demonstrating and explaining their validity; (b) expressing and justifying mathematical generalisations of situations; (c) using various representations to interpolate and extrapolate; (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters.) 11.1.6 Solve non-routine, unseen problems.		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
19	**Probability**	**OPTIONAL ASSESSMENT STANDARDS FOR PAPER 3** **11.4.2** **(a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: P(A and B) = P(A).P(B).** **(b) Use tree and Venn diagrams to solve probability problems (where events are not necessarily independent**		
20	**REVISION**			
21 & 22	**EXAMINATION**			
23	**Probability**	**OPTIONAL ASSESSMENT STANDARDS FOR PAPER 3** **11.4.2** **(a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: P(A and B) = P(A).P(B).** **(b) Use tree and Venn diagrams to solve probability problems (where events are not necessarily independent.**		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
24 & 25	**Pyramids, Cones and Spheres**	11.3.1 Use the formulae for surface area and volume of right pyramids, right cones, spheres and combinations of these geometric objects.		
26 & 27 & 28	**Trigonometry** (including: identities; reduction formulae; solving equations; using the sine, cosine and area rules)	11.3.5 (a) Derive and use the values of the trigonometric functions (in surd form where applicable) of 30º, 45ºand 60º. (b) Derive and use the following identities: (see subject statement) (c) Derive the reduction formulae for: (see Subject Statement) (d) Determine the general solution of trigonometric equations (e) Establish and apply the sine, cosine and area rules. 11.3.6 Solve problems in two dimensions by using the sine, cosine and area rules; and by constructing and interpreting geometric and trigonometric models. 11.1.6 Solve non-routine, unseen problems.		
29 & 30		**OPTIONAL ASSESSMENT STANDARDS FOR PAPER 3** **11.3.2** **(a) Investigate necessary and sufficient conditions for polygons to be similar.** **(b) Prove and use (accepting results established in earlier grades):** • **that a line drawn parallel to one side of a triangle divides the other two sides proportionally (the Mid-point Theorem as a special case of this theorem);** • **that equiangular triangles are similar;** • **that triangles with sides in proportion are similar;** • **the Pythagorean Theorem by similar triangles.**		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
31 & 32 & 33	**Mathematical Modelling with Linear, Quadratic and Exponential Functions** (Including revisiting simple and compound decay)	11.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for functions including: (see Subject Statement) 11.2.1 (a) Demonstrate the ability to work with various types of functions including those listed in the following Assessment Standard. (b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae). 11.2.6 Use mathematical models to investigate problems that arise in real-life contexts: (a) making conjectures, demonstrating and explaining their validity; (b) expressing and justifying mathematical generalisations of situations; (c) using various representations to interpolate and extrapolate; (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features. (Examples should include issues related to health, social, economic, cultural, political and environmental matters.)		
34 & 35	**Probability**	**OPTIONAL ASSESSMENT STANDARDS FOR PAPER 3** **11.4.3** **14 Identify potential sources of bias, error in measurement, potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).** **15 Effectively communicate conclusions and predictions that can be made from the analysis of data.** **11.4.4** **16 Differentiate between symmetric and skewed data and make relevant deductions.** **17**		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
36 37	**Gradients of Curves** (numerical approximation of gradients using small intervals)	11.2.7 Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. 11.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for functions including: (see subject statement) 11.2.1 (a) Demonstrate the ability to work with various types of functions including those listed in the following Assessment Standard. (b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).		
38	**REVISION**			
39-40	**EXAMINATION**			

ANNEXURE 4: EXAMPLE OF A GRADE 12 WORK SCHEDULE FOR MATHEMATICS

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
1	**Logarithms** **Growth and decay**	12.1.2 Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems (e.g. growth and decay see 12.1.4(a)). 12.1.3 (a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b) Correctly interpret sigma notation. (c) Prove and correctly select the formula for and calculate the sum of series, including: $\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}\left[2a + (n-1)d\right]$		
2	**Number Patterns**			
3				
4				

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
5		$\sum_{i=1}^{n} a \times r^{i-1} = \frac{a(r^n - 1)}{r-1}; r \neq 1$ $\sum_{i=1}^{\infty} a \times r^{i-1} = \frac{a}{1-r}$ for $-1 < r < 1$ **Note: Optional Assessment Standard for Paper 3: 12.1.3** **(d) Correctly interpret recursive formulae: (e.g. $T_{n+1} = T_n + T_{n-1}$)**		
6	**Annuities, bond repayments and sinking funds**	12.1.4 (a) Calculate the value of n in the formulae: $A = P(1 \pm i)^n$ (b) Apply knowledge of geometric series to solving annuity, bond repayment and sinking fund problems, with or without the use of the formulae: $F = \frac{x[(1+i)^n - 1]}{i}$ and $P = \frac{x[1-(1+i)^{-n}]}{i}$		
7	**Loans**	12.1.5 Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes). 12.1.6 Solve non-routine, unseen problems.		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
8 - 10	**The Circle**	12.3.3 Use a two-dimensional Cartesian co-ordinate system to derive and apply: (a) the equation of a circle (any centre); (b) the equation of a tangent to a circle given a point on the circle. **NOTE:** learners are expected to know and be able to use as an axiom: "the tangent to a circle is perpendicular to the radius drawn to the point of contact" **Note: Optional Assessment Standards for Paper 3: 12.3.2** **(a) Accept the following as axioms:** *** results established in earlier grades** *** a tangent is perpendicular to the radius, drawn at the point of contact with the circle, and then investigate and prove the theorems of the geometry of circles** *** the line drawn from the centre of a circle, perpendicular to a chord, bisects the chord**		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
		and its converse *** the perpendicular bisector of a chord passes through the centre of the circle** *** the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle** *** angles subtended by a chord at the circle on the same side of the chord are equal and its converse** *** the opposite angles of a cyclic quadrilateral are supplementary and its converse** *** two tangents drawn to a circle from the same point outside the circle are equal in length** *** the angles between a tangent and a chord, drawn to the point of contact of the chord, are equal to the angles which the chord subtends in the alternate chord segments and its converse.** **(b) Use the theorems listed above to:** *** make and prove or disprove conjectures** *** prove riders.**		
11 & 12	**Functions and graphs**	12.2.1 (a) Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard. (b) Demonstrate knowledge of the formal definition of a function		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
13 & 14	**Inverse Functions**	12.2.2 (a) Investigate and generate graphs of the inverse relations of functions, in particular the inverses of: $y = ax + q$ $y = ax^2$ $y = a^x; a > 0$ (b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.		
15	**Inverse Functions and their graphs**	12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above: (a) domain and range; (b) intercepts with the axes; (c) turning points, minima and maxima; (d) asymptotes; (e) shape and symmetry; (f) average gradient (average rate of change); intervals on which the function increases/ decreases.		
16 & 17	**Transformations**	12.3.4 (a) Use the compound angle identities to generalise the effect on the co-ordinates of the point (x; y) after rotation about the origin through an angle. (b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that enlargement preserves shape but not size. 12.3.6 Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models.		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
18	**Data handling**	**Note: Optional Assessment Standards for Paper 3** **12.4.1** **(a) Demonstrate the ability to draw a suitable sample from a population and understand the importance of sample size in predicting the mean and standard deviation of a population.** **(b) Use available technology to calculate the regression function which best fits a given set of bivariate numerical data.** **(c) Use available technology to calculate the correlation co-efficient of a set of bivariate numerical data to make relevant deductions.**		
19	**Linear programming**	12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.		
20 & 21	**REVISION AND EXAMINATION**			
22	**Trigonometry** **Compound angle identities**	12.3.5 Use the following compound angle identities (without derivation): (a) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ (b) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ (c) $\sin 2\alpha = 2\sin\alpha\cos\alpha$ (d) $\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
23 & 24 & 25	**Differential Calculus**	12.2.7 (a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations: • demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient of a function at point; • establishing the derivatives of the following functions from first principles: $f(x) = b$ $f(x) = x$ $f(x) = x^2$ $f(x) = x^3$ $f(x) = \frac{1}{x}$ and then generalise to the derivative of $f(x) = x^n$ (b) Use the following rules of differentiation: $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$ $\frac{d}{dx}\left[kf(x)\right] = k\frac{d}{dx}\left[f(x)\right]$ (c) Determine the equations of the tangents to graphs. (d) Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the x-axis.		

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
26	**Probability**	**Note: Optional Assessment Standards for Paper 3** **12.4.2 Generalise the fundamental counting principle (successive choices from m_1 then m_2 then m_3 options create $m_1 \times m_2 \times m_3$ different combined options) and solve problems using the fundamental counting principle.**		
27	**Linear programming**	12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.		
27	**Data handling**	**Note: Optional Assessment Standards for Paper 3** **12.4.3** **(a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).** **(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.** **12.4.4 Identify data which is normally distributed about a mean by investigating appropriate histograms and frequency polygons.**		
28 & 29	**PREPARATORY EXAMINATION**			

Wk	Content	Learning Outcomes & Assessment Standards	Resources	Assessment
30 - 34	**Trigonometry** **Compound angle identities**	12.3.4 (a) Use the compound angle identities to generalise the effect on the co-ordinates of the point (x; y) after rotation about the origin through an angle. (b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that enlargement preserves shape but not size. 12.3.5 Use the following compound angle identities (without derivation): (a) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ (b) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ (c) $\sin 2\alpha = 2\sin\alpha\cos\alpha$ (d) $\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$		
35 & 36	**REVISION**			
37 - 40	**EXAMINATION**			