

Chapter 7: Trigonometry

Duration: 13,5 hours

BACKGROUND

This chapter consolidates done in trigonometry that applies to 2 and 3 dimensional shapes and objects.

TOPIC OVERVIEW

Learners are expected to

- Revision of solution of triangles.
- Solving problems in 2–dimensions and in 3-dimensions by constructing and interpreting models.

PRIOR KNOWLEDGE

- Pythagoras Theorem
- Trigonometric ratios
- Sine Rule
- Cosine Rule
- Area Rule

SOLUTIONS

Exercise 1

$$1.1 \frac{AC}{\sin 58^\circ} = \frac{6}{\sin 46^\circ}$$

$$AC = \frac{6 \sin 58^\circ}{\sin 46^\circ}$$

$$AC = 7,1 \text{ cm}$$

$$1.2.1 \quad \hat{R} = 180^\circ - (60^\circ + 34^\circ)$$

$$\hat{R} = 86^\circ$$

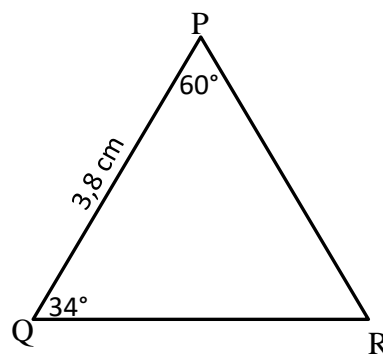
$$1.2.2 \quad \frac{QR}{\sin 60^\circ} = \frac{3,8}{\sin 86^\circ}$$

$$QR = \frac{3,8 \sin 60^\circ}{\sin 86^\circ}$$

$$QR = 3,3 \text{ cm}$$

$$1.3 \quad \hat{W} = 180^\circ - (25^\circ + 100^\circ)$$

$$\hat{W} = 55^\circ$$



$$\frac{UV}{\sin 55^\circ} = \frac{4}{\sin 25^\circ}$$

$$UV = \frac{4 \sin 55^\circ}{\sin 25^\circ}$$

$$UV = 7,75 \text{ cm}$$

$$\frac{UW}{\sin 100^\circ} = \frac{4}{\sin 25^\circ}$$

$$UW = \frac{4 \sin 100^\circ}{\sin 25^\circ}$$

$$UW = 9,32 \text{ cm}$$

$$1.4.1 \quad \hat{L} = 180^\circ - (40^\circ + 32^\circ)$$

$$\hat{L} = 108^\circ$$

$$1.4.2 \quad \frac{NL}{\sin 40^\circ} = \frac{100}{\sin 108^\circ}$$

$$NL = \frac{100 \sin 40^\circ}{\sin 108^\circ}$$

$$NL = 67,6 \text{ m}$$

$$1.4.3 \quad \frac{LP}{\sin 32^\circ} = \frac{100}{\sin 108^\circ}$$

$$LP = \frac{100 \sin 32^\circ}{\sin 108^\circ}$$

$$LP = 55,7 \text{ m}$$

$$1.4.4 \quad \frac{MP}{\sin 45^\circ} = \frac{100}{\sin 99^\circ}$$

$$MP = \frac{100 \sin 45^\circ}{\sin 99^\circ}$$

$$MP = 71,6 \text{ m}$$

1.5.1 In ΔPQR

$$\frac{\sin R}{r} = \frac{\sin Q}{q}$$

$$\sin R = \frac{r \sin Q}{q}$$

$$= \frac{4 \cdot \sin 120}{10}$$

$$\sin R = 0,35$$

$$R = \sin^{-1}(0,35)$$

$$R = 20,5^\circ$$

$$\therefore \hat{P} + \hat{Q} + \hat{R} = 180^\circ \quad (\text{sum } \angle \text{ of } \Delta)$$

$$\hat{P} = 180^\circ - (120^\circ + 20,5)$$

$$\hat{P} = 39,5$$

$$\begin{aligned}
 1.5.2 \quad \frac{QR}{\sin P} &= \frac{PR}{\sin Q} \\
 QR &= \frac{PR \sin P}{\sin Q} \\
 &= \frac{10 \cdot \sin 39,7}{\sin 120} \\
 QR &= 7,38m
 \end{aligned}$$

Exercise 2

$$\begin{aligned}
 2.1 \quad N &= \sqrt{10^2 + 12^2 - 2(10)(12) \cos 102^\circ} \\
 MN &= 17,14 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 2.2 \quad \cos R &= \frac{2,2^2 + 1,4^2 - 1,3^2}{2(2,2)(1,4)} \\
 \hat{R} &= \cos^{-1} \left(\frac{73}{88} \right) \\
 \hat{R} &= 33,9^\circ
 \end{aligned}$$

$$\begin{aligned}
 2.3 \quad CB &= \sqrt{80^2 + 120^2 - 2(80)(120) \cos 78^\circ} \\
 CB &= 129,6 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 2.4.1 \quad AC^2 &= 12^2 + 20^2 - 2(12)(20) \cos 110^\circ \\
 AC &= \sqrt{544 - 480 \cos 110^\circ} \\
 AC &= 26,6 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 2.4.2 \quad \frac{\sin \hat{BAC}}{12} &= \frac{\sin 110^\circ}{26,6} \\
 \hat{BAC} &= \sin^{-1} \left(\frac{12 \sin 110^\circ}{26,6} \right) \\
 \hat{BAC} &= 25
 \end{aligned}$$

$$\begin{aligned}
 2.4.3 \quad 26,6^2 &= 7^2 + 28^2 - 2(7)(28) \cos D \\
 D &= \cos^{-1} \left(\frac{125,44}{392} \right) \\
 \hat{D} &= 71^\circ
 \end{aligned}$$

$$2.5.1 \quad RQ^2 = PR^2 + PQ^2 - 2(PR)(PQ) \cos P$$

$$= 8^2 + 4^2 - 2(8)(4)\cos 130^\circ$$

$$= 64 + 16 - 64\cos 130^\circ$$

$$RQ^2 = 121,138407$$

$$RQ = 11 \text{ m}$$

$$2.5.2 \quad PQ^2 = RP^2 + RQ^2 - 2(RP)(RQ)\cos R$$

$$4^2 = 8^2 + 11^2 - 2(8)(11)\cos R$$

$$-169 = -176\cos R$$

$$\cos R = \frac{169}{176}$$

$$\hat{R} = \cos^{-1}\left(\frac{169}{176}\right)$$

$$\hat{R} = 16,2^\circ$$

$$2.6.1 \quad BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos A$$

$$(5,7)^2 = (11,4)^2 + (7,6)^2 - 2(11,4)(7,6)\cos A$$

$$-155,23 = -173,28$$

$$A = \cos^{-1}\left(\frac{155,23}{173,28}\right)$$

$$A = 26,4$$

$$2.6.2 \quad 90 - 26,4 = 63,6$$

$$\sin 63,6 = \frac{h}{11,4}$$

$$h = 11,4 \sin 63,6$$

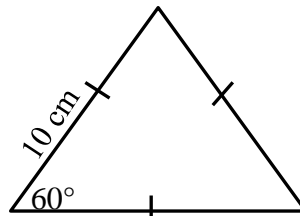
$$h = 10,21 \text{ m}$$

Exercise 3

$$3.1 \quad \text{Area} = \frac{1}{2}bc \sin A$$

$$= \frac{1}{2}(10)(10) \sin 60^\circ$$

$$= 43,3 \text{ cm}^2$$



$$3.2 \quad \text{Area of MNLP} = \frac{1}{2}(4,2)(3) \sin 83^\circ + \frac{1}{2}(2,8)(3,2) \sin 108^\circ$$

$$= 10,5 \text{ cm}^2$$

$$3.3.1 \quad \cos \hat{DAB} = \frac{26^2 + 29^2 - 30^2}{2(26)(29)}$$

$$\hat{A} = \cos^{-1} \frac{26^2 + 29^2 - 30^2}{2(26)(29)} = 65,8^\circ$$

$$3.3.2 \quad \text{Area of ADCB} = \text{Area of triangle ADB} + \text{area of triangle DCB}$$

$$= \frac{1}{2}(26)(29) \sin 65,8^\circ + \frac{1}{2}(30)(18) \sin 80^\circ$$

$$= 609,3 \text{ cm}^2$$

$$\approx 610 \text{ cm}^2$$

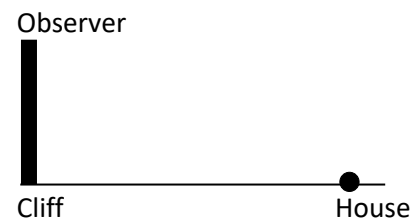
Exercise 4

4.1

$$\tan 12^\circ = \frac{CH}{60}$$

$$CH = 60 \tan 12^\circ$$

$$CH \approx 13 \text{ m}$$



$$4.2 \quad \cos 53^\circ = \frac{6}{BC}$$

$$BC = \frac{6}{\cos 53^\circ}$$

$$BC \approx 10 \text{ m}$$

4.3

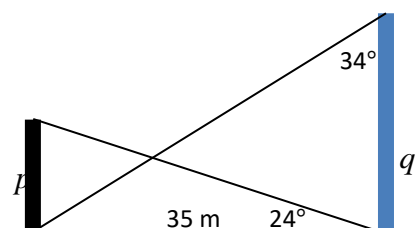
Building p :

$$\tan 24^\circ = \frac{p}{35}$$

$$p = 35 \tan 24^\circ$$

$$p = 15,58$$

building p is 15,58 m



Building q :

$$\tan 34^\circ = \frac{35}{q}$$

$$q = \frac{35}{\tan 34^\circ}$$

$$q = 51,88 \text{ m}$$

building q is 51,88 m

$$4.4 \quad on = 10 - 4 = 6 \text{ m}$$

$$\tan N\hat{Q}O = \frac{6}{20}$$

$$N\hat{Q}O = \tan^{-1} \frac{6}{20}$$

$$N\hat{Q}O = 16,7^\circ$$

Exercise 5

$$5.1.1 \quad \tan 40^\circ = \frac{3}{LB}$$

$$LB = 3 \tan 40^\circ$$

$$LB = 3,6 \text{ m}$$

$$5.1.2 \quad AB^2 = 5,2^2 + 3,6^2 - 2(5,2)(3,6) \cos 113^\circ$$

$$AB = \sqrt{54,62897 \dots}$$

$$AB = 7,4 \text{ m}$$

$$5.1.3 \quad \text{Area of } \Delta ABC = \frac{1}{2} \times 5,2 \times 3,6 \sin 113^\circ \\ = 8,28 \text{ m}^2$$

$$5.2.1 \quad \text{Area of } \Delta ABC = \frac{1}{2} \times 5 \times 5 \sin 50^\circ \\ = 9,58 \text{ cm}^2$$

$$5.2.2 \quad \hat{A} = \hat{C} = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

$$\frac{AC}{\sin 50^\circ} = \frac{5}{\sin 65^\circ}$$

$$AC = \frac{5 \sin 50^\circ}{\sin 65^\circ}$$

$$AC = 4,2 \text{ cm}$$

OR

$$AC^2 = 5^2 + 5^2 - 2(5)(5) \cos 50^\circ$$

$$AC = \sqrt{17,86061952}$$

$$AC = 4,2 \text{ cm}$$

$$5.2.3 \quad \tan 25^\circ = \frac{FC}{4,2}$$

$$FC = 4,2 \tan 25^\circ$$

$$FC = 2 \text{ m}$$

$$5.3.1 \quad \sin 48^\circ = \frac{50}{RN}$$

$$RN = \frac{50}{\sin 48^\circ}$$

$$RN = 67,3 \text{ m}$$

$$5.3.2 \quad \frac{67,3}{\sin 55^\circ} = \frac{MN}{\sin 71^\circ}$$

$$MN = \frac{67,3 \sin 55^\circ}{\sin 71^\circ}$$

$$MN = 77,7 \text{ m}$$

$$5.3.3 \quad SN = \sqrt{67,3^2 - 50^2} = 45,0 \text{ m}$$

$$\text{Area of } \Delta RSN = \frac{1}{2} \times SN \times 50$$

$$= 0,5 \times 45,50$$

$$= 1 \ 125 \text{ m}^2$$

$$5.4.1 \quad \sin 18^\circ = \frac{7}{PB}$$

$$PB = \frac{7}{\sin 18^\circ} = 22,65 \text{ m}$$

$$5.4.2 \quad \frac{18}{PA} = \cos 23^\circ$$

$$PA = \frac{18}{\cos 23^\circ}$$

$$PA = 19,55 \text{ m}$$

$$5.4.3 \quad AB^2 = 22,5^2 + 19,55^2 - 2(22,5)(19,55) \cos 42^\circ$$

$$AB^2 = 237,0847954$$

$$AB = 15,4 \text{ m}$$

Revision exercise

$$1. \quad \frac{XY}{\sin 21^\circ} = \frac{12}{\sin 132^\circ}$$

$$XY = \frac{12 \sin 21^\circ}{\sin 132^\circ}$$

$$XY = 5,79 \text{ cm}$$

2.

$$\begin{aligned} \text{a) } \hat{A}BC &= 180^\circ - 135^\circ \\ &= 45^\circ \end{aligned}$$

$$\begin{aligned} \text{b) } AC^2 &= 20^2 + 15^2 - 2(20)(15) \cos 45 \\ AC^2 &= 200,7359313 \\ AC &= 14,17 \end{aligned}$$

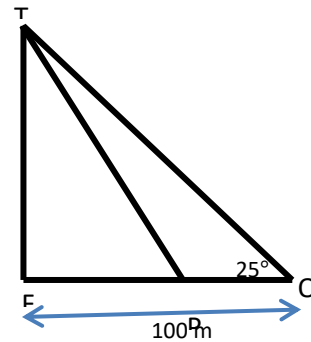
$$\begin{aligned} \text{c) } \frac{20}{\sin \hat{A}CB} &= \frac{14,17}{\sin 45^\circ} \\ \hat{A}CB &= \sin^{-1} \left(\frac{20 \sin 45^\circ}{14,17} \right) \\ \hat{A}CB &= 86,41 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{3. Area of } \Delta ABC &= \frac{1}{2} \times 5 \times 6 \times \sin 48^\circ \\ &= 11,15 \text{ cm}^2 \end{aligned}$$

4.

$$\begin{aligned} \text{a) } \tan 25^\circ &= \frac{TF}{100} \\ TF &= 100 \tan 25^\circ \\ TF &= 46,6 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{b) } \tan \hat{P} &= \frac{46,6}{60} \\ \hat{P} &= 37,8^\circ \end{aligned}$$



$$\begin{aligned} \text{c) Angle of depression from } &= 180^\circ - (90^\circ + 37,8^\circ) \\ &= 52,2^\circ \end{aligned}$$

5.

a)

In ΔABC , $\hat{D} = 65^\circ$ (sum of angles in a Δ)

$$\frac{AC}{\sin 65^\circ} = \frac{158}{\sin 25^\circ}$$

$$\tan 25^\circ = \frac{158}{AC}$$

$$\begin{aligned} AC &= \frac{158}{\tan 25^\circ} \\ AC &= 338,83 \text{ m} \end{aligned}$$

$$AC = 338,83 \text{ m}$$

$$\text{b) In } \Delta ACB, BC^2 = 338,83^2 + 1500^2 - 2(338,83)(1500) \cos 30^\circ$$

$$BC^2 = 1484499,606$$

$$BC = 1218,4 \text{ m}$$

$$\text{c) In } \Delta BCD \tan \theta = \frac{DC}{BC}$$

$$\tan \theta = \frac{158}{1218,4}$$

$$\theta = \tan^{-1} \left(\frac{158}{1218,4} \right)$$

$$\theta = 7,4^\circ$$

$$\text{d) Area of } \Delta ABC, = 0,5 \times 338,83 \times 1500 \sin 30^\circ$$

$$= 127\,061,25 \text{ m}^2$$

$$\text{e) } AD^2 = 338,83^2 + 158^2$$

$$AD = 373,86 \text{ m}$$

$$BD^2 = 1218,4^2 + 158^2$$

$$BD = 1228,6 \text{ m}$$

$$1500^2 = 373,86^2 + 1228,6^2 - 2(373,86)(1228,6) \cos \widehat{ADB}$$

$$\cos \widehat{ADB} = \frac{-600770,7404}{918648,792}$$

$$\widehat{ADB} = 130,8^\circ$$